Generic Σ^1_3 Absoluteness

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In this article we study the strength of Σ^1_3 absoluteness (with real parameters) in various types of generic extensions, correcting and improving some results from [2]. We shall also make some comments relating this work to the bounded forcing axioms BMM, BPFA and BSPFA.

The statement "Σ^1_3 absoluteness holds for ccc forcing" means that if a Σ^1_3 formula with real parameters has a solution in a ccc set-forcing extension of the universe V, then it already has a solution in V. The analogous definition applies when ccc is replaced by other set-forcing notions, or by class-forcing.

Theorem 1 *([1]) Σ^1_3 absoluteness for ccc has no strength; i.e., if ZFC is consistent then so is ZFC + Σ^1_3 absoluteness for ccc.*

The following results concerning (arbitrary) set-forcing and class-forcing can be found in [2].

Theorem 2 *(a) (Feng-Magidor-Woodin) Σ^1_3 absoluteness for arbitrary set-forcing is equiconsistent with the existence of a* reflecting cardinal*, i.e. a regular cardinal κ such that $H(\kappa)$ is Σ_2-elementary in V.*
(b) Σ^1_3 absoluteness for class-forcing is inconsistent.

We consider next the following set-forcing notions, which lie strictly between ccc and arbitrary set-forcing: proper, semiproper, stationary-preserving and ω_1-preserving.

Using a variant of an argument due to Goldstern-Shelah (see [5]), we show the following.

Theorem 3 Σ^1_3 *absoluteness for semiproper forcing has no strength.*

Proof. By an ω_1-iteration P_0 of semiproper forcing with revised countable support, produce a generic G_0 such that $L[G_0]$ satisfies semiproper absoluteness for Σ^1_3 formulas with real parameters in L. This is possible as there are only ω_1 reals in L and semiproperness is preserved through iteration with revised countable support. We can assume that P_0 has cardinality ω_1 in $L[G_0]$, as if necessary we can follow P_0 by a Lévy collapse with countable conditions to ω_1. Thus we have $L[G_0] = L[X_0]$, where X_0 is a subset of ω_1.

Now repeat the above over the model $L[X_0]$, guaranteeing with a semiproper revised countable support iteration of length ω_1 that semiproper absoluteness holds in $L[X_0, X_1]$ for Σ^1_3 formulas with real parameters from $L[X_0]$, where X_1 is a subset of ω_1. Repeat this for ω_1 stages, producing $L[X_i, i < \omega_1]$, a model where semiproper absoluteness holds for Σ^1_3 formulas with real parameters in $\bigcup_{i<\omega_1} L[X_j, j < i]$.

Claim. Every real in $L[X_i, i < \omega_1]$ belongs to $L[X_j, j < i]$ for some $i < \omega_1$.

Proof of Claim. If R is a real in $L[X_i, i < \omega_1]$ then R belongs to a countable, sufficiently elementary submodel M of $L[X_i, i < \omega_1]$, as well as to the transitive collapse $\bar{M}$ of M. But $\bar{M}$ is equal to $L_\alpha[X_i \cap \beta, i < \beta]$, where α is the ordinal height of $\bar{M}$ and β is the ω_1 of $\bar{M}$. It follows that R belongs to $L[X_i, i < \beta]$. This proves the Claim.

Thus $L[X_i, i < \omega_1]$ is a model of semiproper Σ^1_3 absoluteness for formulas with arbitrary real parameters, as desired. □

Remark. In the previous proof, we can begin with any L-cardinal κ of L-cofinality ω_1^L such that L_κ is Σ_2-elementary in L and by "bookkeeping" perform a single semiproper ω_1^L-iteration with revised countable support guaranteeing Σ^1_3 semiproper absoluteness, where at each stage $i < \omega_1^L$, the iteration up to stage i belongs to L_κ. Thus the final model of Σ^1_3 absoluteness for semiproper forcing can be of the form $L[X]$, where $X \subseteq \omega_1^L$ is generic over L for a forcing of L-cardinality κ.

We say that ω_1 is *inaccessible to reals* if and only if for every real x, ω_1 is inaccessible in $L[x]$; equivalently, ω_1 of $L[x]$ is countable for each real x. In the presence of this additional assumption, Σ^1_3 absoluteness for semiproper (and even proper) forcing has the full strength of Σ^1_3 absoluteness for arbitrary set-forcings:

Theorem 4 Σ^1_3 *absoluteness for proper* + ω_1 *is inaccessible to reals has the consistency strength of a reflecting cardinal.*

Proof. One direction is easy: If κ is reflecting in L then after a Lévy collapse with finite conditions to make κ equal to ω_1, one obtains a model in which ω_1 is inaccessible to reals and in which Σ^1_3 absoluteness holds for arbitrary set-forcings. (See either [3] or Theorem 3 of [2].)

Conversely, assume that Σ^1_3 absoluteness holds for proper forcings and that ω_1 is inaccessible to reals. We may assume that ω_1 is not Mahlo in L, else the ZFC-model L_{ω_1} satisfies that there exists a reflecting cardinal. We will show that ω_1 is reflecting in L. The proof is a refinement of the proof of Theorem 4 of [2].

Let κ denote ω_1 of V. It suffices to show that if x belongs to L_κ, φ is a formula and for some L-cardinal $\lambda \geq \kappa$, $L_\lambda \vDash \varphi(x)$ then there is such a $\lambda < \kappa$ with $x \in L_\lambda$.

Assume that $L_\lambda \vDash \varphi(x)$, $\lambda \geq \kappa$ is an L-cardinal and let R be a real coding x. We may assume that $0^\#$ does not exist, as otherwise ω_1 is surely reflecting in L. Then in a countably-closed set-forcing extension there is $A \subseteq \omega_1$ such that:

a. $\lambda < \omega_2$ and in fact λ is less than the height of the least transitive model of ZF^- containing A and ω_1.
b. Every subset of ω_1 belongs to $L[A]$ and in particular $\omega_2 = \omega_2$ of $L[A]$.

A is obtained as follows: Let $\delta > \lambda$ be a singular strong limit cardinal of uncountable cofinality. Since $0^\#$ does not exist, we have $\delta^+ = \delta^+$ of L and $2^\delta = \delta^+$. Now collapse δ to ω_1 using countable conditions. This produces $A_0 \subseteq \omega_1$ such that ω_2 of $L[A_0] = \delta^+$ of L and in this extension $2^{\omega_1} = \omega_2$. In this model let $B \subseteq \omega_2$ code all subsets of ω_1 and using the fact that ω_2 is a successor cardinal of L, code B by $A \subseteq \omega_1$ via a countably closed almost disjoint forcing. In the extension, every subset of ω_1 belongs to $L[A]$.

In $L[A]$ the following holds:

$(*)$ If $L_\alpha[A]$ is a model of ZF^-, $\alpha > \omega_1$ then $L_\alpha[A] \models$ There is an L-cardinal λ such that $\varphi(x)$ holds in L_λ, where x is the set coded by R.

Now add $A^* \subseteq \omega_1$ with the following improved version of $(*)$:

$(**)$ If $L_\alpha[A^* \cap \gamma]$ is a model of ZF^- where $\alpha > \gamma$ and γ is the ω_1 of $L_\alpha[A^* \cap \gamma]$ then $L_\alpha[A^* \cap \gamma] \models$ There is an L-cardinal λ such that $\varphi(x)$ holds in L_λ, where x is the set coded by R.

A^* is obtained as follows. Let P be the set of $p : \gamma(p) \to 2$, $\gamma(p) < \omega_1$, such that:

$(***)$ For all $\gamma \leq \gamma(p)$ and all α, if $L_\alpha[A \cap \gamma, p \restriction \gamma]$ is a model of ZF^- where $\alpha > \gamma$ and γ is the ω_1 of $L_\alpha[A \cap \gamma, p \restriction \gamma]$ then $L_\alpha[A \cap \gamma, p \restriction \gamma] \models$ There is an L-cardinal λ such that $\varphi(x)$ holds in L_λ, where x is the set coded by R.

A P-generic adds a function $F : \omega_1 \to 2$ such that $A^* = \{2\beta \mid \beta \in A\} \cup \{2\beta + 1 \mid F(\beta) = 1\}$ satisfies $(**)$, since this is guaranteed for countable γ by the definition of P and for $\gamma = \omega_1$ by $(*)$. It remains to show:

Lemma 5 *P is proper.*

Proof. This is a special case of an argument of [10], which was put to excellent use in [8]; a more general version of this particular proof can be found in [11].

We must show that for CUB many countable $N \prec L_{\omega_2}[A]$, each condition p in N can be extended to a condition q such that q forces each name in N for an ordinal to equal an ordinal of N. We take all countable $N \prec L_{\omega_2}[A]$ which have A and R as elements. Suppose that p belongs to N and let N be isomorphic to $\bar{N} = L_\gamma[A \cap \beta]$, where β is the ω_1 of $\bar{N}$. As N contains a witness to the non-Mahloness of ω_1 in L, it follows that β is singular in L, and therefore γ is not an L-cardinal. Let δ be least so that γ is collapsed in L_δ. Notice that $(***)$ does hold when γ of $(***)$ is equal to β and α of $(***)$ is at most γ, by the elementarity of N in $L_{\omega_2}[A]$. $(***)$ also holds when γ of $(***)$ is equal to β and α of $(***)$ is between γ and δ, as in this case any L-cardinal of L_γ is also an L-cardinal of L_α. Thus it suffices to build q extending p of length β, as the union of conditions of length less than β, so that β is collapsed in $L_\delta[A\cap\beta, q]$, for then $(***)$ is vacuous when γ of $(***)$ is equal to β and α of $(***)$ is at least δ.

As γ is collapsed to β in L_δ, we can write $L_\gamma[A\cap\beta]$ as the union of a continuous elementary chain of elementary submodels of $L_\gamma[A\cap\beta]$ of length β, where each model is countable in $L_\delta[A\cap\beta]$ and the chain itself belongs to $L_\delta[A\cap\beta]$. Let C be the set of intersections of the models of this chain with β, a CUB subset of β. Then we can choose an ω-sequence $p = p_0 \geq p_1 \geq \cdots$ of conditions below p such that each p_i belongs to N, each name for an ordinal in N is forced by some p_i to equal an ordinal of N and if q is the union of the p_i's, and $q(\eta) = 0$ on C, except for a cofinal subset of C of ordertype ω. Then β is collapsed in $L_\delta[A\cap\beta, q]$, as desired. $\Box$ (Lemma 5).

Now we code A by a real S. As ω_1 is not Mahlo in L, ω_1 is *reshaped* in the sense that for some $B\subseteq\omega_1$, α countable $\to$ α countable in $L[B\cap\alpha]$. Using this, we can choose reals R_α, $\alpha<\omega_1$ so that R_α can be defined uniformly in $L[B\cap\alpha]$, and use these reals to code B, A^* and R by a real S using a ccc almost disjoint coding. As $A^*\cap\omega_1^{L_\alpha[S]}$ is definable in $L_\alpha[S]$ for each $\alpha<\omega_1$, we get:

For all α, if $L_\alpha[S]$ is a model of $\mathrm{ZF}^- + \omega_1$ exists, then $L_\alpha[S] \vDash$ There is an L-cardinal λ such that $\varphi(x)$ holds in L_λ, where x is the set coded by R.

This is a Π^1_2 condition on S, and therefore by our assumption that Σ^1_3 absoluteness holds for proper forcing, there is such an S in V. By our assumption that ω_1 is inacessible to reals, $L_{\omega_1}[S]$ satisfies that ω_1 exists, and therefore there is $\lambda<\omega_1$ such that $L_\lambda\vDash\varphi(x)$ and $L_{\omega_1}\vDash\lambda$ is an L-cardinal. It follows that λ is an L-cardinal, and therefore we have completed the proof that ω_1 is reflecting in L. $\Box$

Remark. Σ^1_3 absoluteness for proper $+\,\omega_1$ inaccessible to reals actually implies that ω_1 is reflecting to reals (i.e., ω_1 is reflecting in $L[R]$ for each real R). This is because Lemma 5 only requires the assumption that ω_1 is not *remarkable* to reals (a property stronger than reflection, see [7]) and under the same assumption, one can reshape ω_1 using a proper forcing.

Corollary 6 *Σ^1_3 absoluteness for ω_1-preserving forcing is equiconsistent with a reflecting cardinal.*

Proof. By Corollary 1 of [2], the above form of absoluteness implies that ω_1 is inaccessible to reals. Therefore by the previous Theorem, we get the consistency strength of a reflecting cardinal. $\Box$

Using a recent technique of Ralf Schindler [9], we next compute the consistency strength of Σ^1_3 absoluteness for stationary-preserving forcing:

Lemma 7 *Suppose that Σ^1_3 absoluteness holds for stationary-preserving forcings. Then ω_1 is inaccessible to reals.*

Proof. We first need the following, which was proved independently by Schindler (see [8]).

Lemma 8 *If $A^\#$ does not exist for some set of ordinals A then every set of ordinals is constructible from a real in a stationary-preserving forcing extension.*

Proof. As in the proof of Theorem 4 (see the construction in that proof of the set A), we can produce $A \subseteq \omega_1$ by a countably-closed forcing so that in the extension $H(\omega_2) = L_{\omega_2}[A]$ and the given set of ordinals belongs to $H(\omega_2)$. Let P be the "reshaping forcing", whose conditions are $p : |p| \to 2$, $|p| < \omega_1$ such that for all $\alpha \leq |p|$, α is countable in $L[A \cap \alpha, p \restriction \alpha]$. We show that P is stationary-preserving. Given this, we can then apply a ccc forcing to code A, G by a real (where $G \subseteq \omega_1$ is P-generic), resulting in an extension in which the given set of ordinals is constructible from a real.

We show now that P is stationary-preserving. Given $p \in P$, a stationary $X \subseteq \omega_1$ and a name σ for a CUB subset of ω_1, let C be a CUB subset of ω_1 such that:

1. $\alpha \in C$, $\beta < \alpha \to p \in L_\alpha[A]$ and every $q \leq p$ in $L_\alpha[A]$ has an extension $r \in L_\alpha[A]$ such that $r \Vdash \beta^* \in \sigma$ for some β^* between β and α.
2. $\alpha \in C \to C \cap \alpha$ belongs to $L[A \cap \alpha]$.

C is easily constructed, by choosing $L_\gamma[A]$ to contain p and σ, taking a chain $\langle M_i \mid i < \omega_1 \rangle$ of countable elementary submodels of $L_\gamma[A]$ with $p, \sigma, A \in M_0$ and setting $C = \{M_i \cap \omega_1 \mid i < \omega_1\}$.

Now choose $\alpha \in \text{Lim}\, C \cap X$ and let D be any cofinal subset of $C \cap \alpha$ of ordertype ω. Using property 1 above, we can choose q with domain α to be the union of conditions q_i below p so that $q_i \Vdash \beta_i \in \sigma$, where the β_i are unbounded in α and $\{\beta \in C \cap \alpha \mid q(\beta) = 1\}$ is a final segment of D. By property 2 above, D belongs to $L[A \cap \alpha, q]$, and therefore α is countable in $L[A, q]$, establishing that q is a condition. As q forces that $\sigma \cap \alpha$ is unbounded in α, q also forces that α belongs to σ; as α belongs to X, we have $q \Vdash X \cap \sigma \neq \emptyset$, as desired. □ (Lemma 8)

Now to prove Lemma 7, suppose that ω_1 is not inaccessible to reals. In particular the hypothesis of Lemma 8 holds, so in a stationary-preserving set-generic extension, $V = L[R]$ for some real R. As the real R plays no role in the arguments below, we assume that R equals 0. For any $A \subseteq \omega_1$ consider the function $f_A : \omega_1 \to \omega_1$ defined by

$f_A(\alpha) =$ the least β such that α is countable in $L_{\beta+1}[A \cap \alpha]$.

We say that A is *faster than* B iff $f_A < f_B$ on a CUB.

Lemma 9 *(Ralf Schindler) For any A there is a faster B in a further stationary-preserving forcing extension.*

Proof. Consider the forcing P whose conditions are pairs (b, c) where:

c is a countable closed subset of ω_1.
$b : \max c \to 2$.
For all $\alpha \in c$, α is countable in $L_{f_A(\alpha)}[b \restriction \alpha]$.

Any condition can be extended so as to increase $\max c$ above any given countable ordinal: Given (b, c) there are limit ordinals $\alpha > \max c$ with $f_A(\alpha) > \alpha$. We obtain a condition by adding α to c and extending b to b' of length α so that α is countable in $L_{\alpha+1}[b']$, using an ω-sequence cofinal in α. Thus if G is P-generic then $B = \bigcup\{b \mid (b, c) \in G \text{ for some } c\}$ is faster than A, as witnessed by the CUB set $C = \bigcup\{c \mid (b, c) \in G \text{ for some } b\}$. It remains only to show that P is stationary-preserving.

Suppose that $(b, c) \in P$, S is stationary and σ is a name for a CUB. Let C be a CUB set such that:

If α belongs to C then p belongs to L_α and for each $\beta < \alpha$, each $q \leq p$ in L_α has an extension $r \in L_\alpha$ such that $r \Vdash \beta^* \in \sigma$ for some β^* between β and α.

C is easily constructed, by choosing L_γ to contain p, σ, A, taking a continuous chain $\langle M_i \mid i < \omega_1 \rangle$ of countable elementary submodels of L_γ with $p, \sigma, A \in M_0$ and setting $C = \{M_i \cap \omega_1 \mid i < \omega_1\}$.

Now choose L_γ to contain A, C and $\alpha \in \text{Lim } C \cap S$ such that $\alpha = M \cap \omega_1$ for some elementary submodel M of L_γ containing A and C. Let D be any cofinal subset of $C \cap \alpha$ of ordertype ω. We can extend p to a descending sequence of conditions $q_i = (b_i, c_i)$ of height less than α so that $q_i \Vdash \beta_i \in \sigma$, where the β_i are unbounded in α and if $b = \cup_i b_i$ then $\{\beta \in C_0 \cap \alpha \mid b(\beta) = 1\}$ is a final segment of D. It follows that D belongs to $L_{f_A(\alpha)}[b]$ and therefore we obtain a condition $q = (b, c)$ where $c = (\cup_i c_i) \cup \{\alpha\}$. Then q extends p and forces that α belongs to $S \cap \sigma$, as desired. □ (Lemma 9)

Finally, set $A_0 = \emptyset$. By Lemma 9 there is A_1 which is faster than A_0 in a stationary-preserving forcing extension. A_1, together with a CUB set C_1 witnessing that A_1 is faster than A_0, can be coded by a real R_1 via a ccc forcing; then R_1 satisfies the Π^1_2 condition

For all $\alpha < \omega_1$, $f_{R_1}(\alpha) < f_\emptyset(\alpha)$ for all α in the CUB set coded by R_1.

By Σ^1_3 absoluteness for stationary-preserving forcings, there is such a real R_1 in the ground model. But we can repeat this, obtaining R_{n+1} which is faster than R_n, for each n. Thus $f_{R_{n+1}} < f_{R_n}$ on a CUB for each n, a contradiction. □ (Lemma 7)

Corollary 10 Σ^1_3 *absoluteness for stationary-preserving forcing has the strength of a reflecting cardinal.*

Proof. By Theorem 4 and Lemma 7. □

Using a refinement of the technique of [6], David Schrittesser has shown the following:

Theorem 11 *(Schrittesser, see [11])* Σ^1_3 *absoluteness for ccc +* ω_1 *inaccessible to reals has the consistency strength of a lightface* Σ^1_2 *reflecting cardinal, i.e., an inaccessible cardinal* κ *such that every* Σ^1_2 *sentence with parameters from* H_κ *which is true in* H_κ *is also true in* H_γ *for some* $\gamma < \kappa$.

The Bounded Forcing Axioms

Goldstern-Shelah (see [5]) introduced the *bounded forcing axioms*, which include BMM, BPFA and BSPFA. By the technique of [1], these are equivalent to $\Sigma_1(H(\omega_2))$ absoluteness (with parameters from $H(\omega_2)$) for stationary-preserving, proper and semiproper forcing, respectively. Also, MA (Martin's axiom) is equivalent to $\Sigma_1(H(\omega_2))$ absoluteness (with parameters from $H(\omega_2)$) for ccc forcing. Note that $\Sigma_1(H_{\omega_2})$ absoluteness implies Σ^1_3 absoluteness.

It is shown in [5] that BPFA and BSPFA have the strength of a reflecting cardinal. However unlike BMM, these axioms do not imply that ω_1 is inaccessible to reals; indeed one has:

Theorem 12 *(a) ([6]) MA + ω_1 inaccessible to reals has the strength of a weakly compact cardinal.*
(b) ([7]) BPFA + ω_1 inaccessible to reals has the strength of a remarkable cardinal with a reflecting cardinal above.

When ω_1 is *not* inaccessible to reals, then these axioms reduce to their Σ^1_3 counterparts:

Theorem 13 *Suppose that ω_1 is not inaccessible to reals.*
(a) MA is equivalent to Σ^1_3 absoluteness for ccc + every subset of ω_1 is constructible from a real.
(b) BPFA is equivalent to Σ^1_3 absoluteness for proper + every subset of ω_1 is constructible from a real.
(c) BSPFA is equivalent to Σ^1_3 absoluteness for semiproper + every subset of ω_1 is constructible from a real.
Moreover (a) only needs the assumption that ω_1 is not weakly compact to reals and (b),(c) only need that ω_1 is not remarkable to reals.

Proof. We prove (a). As ω_1 is not inaccessible to reals, every subset of ω_1 can be coded by a real via a ccc almost disjoint forcing. This gives the implication from left to right. Conversely, assume Σ^1_3 absoluteness for ccc + every subset of ω_1 is constructible from a real. We want to establish $\Sigma_1(H(\omega_2))$ absoluteness with parameters from $H(\omega_2)$ for ccc forcings. As every subset of ω_1 is constructible from a real, it is sufficient to show that for each formula φ and real R, if in a ccc extension there is $\alpha < \omega_2$ such that $L_\alpha[R] \vDash \varphi(R, \omega_1)$ then this holds in V. We may assume that α is less than the height of the least ZF^- model containing R and ω_1, by further coding into a real. Then by reflection, there is a CUB subset C of ω_1 such that $L_\beta[R] \vDash \varphi(R, \gamma)$ whenever γ belongs to C and is the ω_1 of the ZF^- model $L_\beta[R]$. By further ccc coding, we may assume that whenever γ is the ω_1 of some countable ZF^- model $L_\beta[R]$ then γ belongs to C. Therefore in a ccc extension there is a real R obeying the Π^1_2 property:

$(*)$ For every ZF^- model $L_\beta[R]$ with $\gamma = \omega_1$ of $L_\beta[R]$, there is $\alpha < \beta$ such that $L_\alpha[R] \vDash \varphi(R, \gamma)$.

By hypothesis there is such a real R in V. If we apply $(*)$ to $\beta = \omega_2$ then we get $L_\alpha[R] \vDash \varphi(R, \omega_1)$, for some $\alpha < \omega_2$, as desired.

The same argument proves (b) and (c), given the assumption that ω_1 is not inaccessible to reals. For the last statement of the Theorem: By [6], if ω_1 is not weakly compact to reals then every subset of ω_1 is constructible from a real in a ccc forcing extension, and therefore under MA this holds in V. Similarly, if ω_1 is not remarkable to reals then by [7], every subset of ω_1 is constructible from a real in a proper forcing extension, and therefore under BPFA this holds in V. □

Open Question. What is the strength of BSPFA + ω_1 is inaccessible to reals? What is the strength of BMM?

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