EINFÜHRUNG IN DIE MATHEMATISCHE LOGIK WS 2015

VERA FISCHER

The course is an introduction to mathematical logic. We will cover the theorems of Gödel for Incompleteness, as well as some introductory model and set theory. Some knowledge from the course of Dr. Moritz Müller, "Grundbegriffe der mathematischen Logik ", see

http://www.logic.univie.ac.at/∼muellem3/teaching.html.

will be assumed. Our main references are listed below. Detailed information on the material covered during the semester will be regularly given here.

The exam will be oral. If you want to take the exam send me an e-mail at least 2 days in advance. Alternatively, you can take the exam on the 17th of February 2016 (Wednesday) at 10:00am.

The lectures will be taking place **Mondays at 8:00am** and **Fridays at 9:00am** in the KGRC, lecture room 101.

Lecture 1, 01.10.: We covered Section 1.1 of [2] up to and including Lemma 1.1.30.

Lecture 2, 05.10.: We proved Lemma 1.1.32 from section 1.1. and proceeded with section 1.2 up to (and including) Theorem 1.2.33.

Lecture 3, 09.10.: We defined the notion of complete subsets of the sentential language and considered examples; we covered most of section 1.3 from [2] including definitions 1.3.37 and 1.3.41. We concluded with some examples of valid formulas.

Lecture 4, 12.10.: We continued with section 1.3 of [2] up to and including the proof of Lemma 1.65. From Section 1.4 we covered the material up to and including Remark 1.4.12.

Lecture 5, 16.10.: We discussed the theorems of Soundness, Deduction, Generalizations, Intorduction of Universal Quantifier, Intorduction of Existential Quantifier and introduced the notions of consistency and enumerability.

Lecture 6, 19.10.: We proved the Compactness theorem of first order logic, the existence of complete simple extnesions of consistent theories, the theorem on constants, the existence of Henkin extensions for consistent theories and proceeded with the proof of the completeness theorem up to and includig Lemma 2.2.11.

Lecture 7, 23.10.: We finished the proof of the Completeness theorem. Introduced the notions of isomorphic models and definable sets, and proceeded with chapter 3 of [2] up to Fact 3.1.4.

Lecture 8, 30.10.: Section 3.1 up to 3.1.5.

Lectrue 9, 06.11.: We proceeded with section 3.1. and outlined a proof of the joint consistency theorem (Theorem 3.1.21).

Lecture 10, 09.11.: We proved the interpolation theorem (Thm 3.1.26), intorduced ultraproducts and proved semantic compactness, i.e. Thm 3.2.12. We concluded by introducing non-standard models of arithmetic (Def. 2.3.4.)

Lecture 11, 13.11.: We showed that non-standard models of arithmetic exists and that there are uncountably many non-isomorphic countable models of arithmetic (Theorem 2.3.19). We continued with section 3.3.(Types and Countable Models) and concluded with a proof of Fact 3.3.11.

Lecture 12, 16.11.: We continued with section 3.3 and discussed two complete theories, one with exactly and the other with exactly three (up to isomorphism) countable models. We introduced saturated models and proved Theorem 3.3.29.

Lecture 13, 20.11.: We gave a characterizaton of theories which have saturated models and started our discussion of atomic models (the material corresponds to p.170 - p.173 in [2]).

Lecture 14, 23.11.: We continued with our discussion of complete theories with atomic models, gave a characterization of categorical theories and concluded with Theorem 3.3.48 of [2].

Lecture 15, 27.11.: We started with our study of Peano Arithmetic, intorduced the axioms and covered the material up to and including Example 4.3.2.

Lecture 16, 30.11.: up to and including page 206.

Lecture 17, 04.12.: Gödel Numbers

Lecture 18, 07.12.: Gödel's Incompleteness Theorem, Gödel's Self-Referential Lemma and Corollary 4.8.2, the Undefinability of Truth.

Lecture 19, 11.12.: Bounded, Σ^0_1 and Π^0_1 formulas, recursively enumerable and recursive sets; We concluded with a proof of Thm 4.10.7, that the set of Gödel numbers of provable closed Σ^0_1 formulas is r.e. but not recursive.

Lecture 20, 14.12.: We started our discussion of set theory. Defined the axiomatic system of ZFC, and considered in detail Axioms 1- 6 (Extensionality, Foundation, Comprehension, Pairing, Union and Replacement). Defined and justified empty set, some natural numbers, cartesian productrs. Made a review of some notions regarding relations and functions.

Lecture 21, 18.12.: We defined the notions of an ordinal number, as well as ordinal multiplicaiton and ordinal addition.

Lecture 22, 08.01.: We discussed transfinite induction and transfinite recursion on the ordinals, and proved the Schröder-Bernstein Theorem.

Lectures 23 and 24.: Dr. Müller covered sections I.11 and I.12 from [3].

Lectures 25 and 26.: We finished the discussion of cardinal arithmetic (section I.13 of [3]) and proceded with the axiom of foundation up to Lemma I.14.5 from [3].

Lecture 27.: We finished our discussion of the axiom of foundation (section 1.14 of [3]).

Lecture 28, 29.01.: Definitional extensions, Loeb Axioms, Gödel's Second Theorem of Incompleteness for ZFC.

References

[1] H.-D. Ebbinghaus, J. Flum, W. Thomas *Einführung in die mathematische Logik* Springer, 2007.
[2] M. Goldstern, H. Judah *The incompleteness phenomenon*, A. K. Peters, 1998.
[3] K. Kunen *The Foundations of Mathematics*, Studies in Logic (London), 19. Mathematical Logic and Foundations. College Publications, London, 2009. viii+251 pp.
[4] K. Kunen *Set theory*, Studies in Logic (London), 34. College Publications, London, 2011, viii+401 pp.

Kurt Gödel Research Center, University of Vienna, Währingerstrasse 25, 1090 Vienna, Austria
E-mail address: vera.fischer@univie.ac.at