ESI Workshop on Large Cardinals and Descriptive Set Theory

Vienna, June 14–27, 2009

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Some descriptive set theory related to the Lebesgue density theorem

The Lebesgue density theorem says that if $A \subseteq {}^{\omega}2$ is measurable, then A is almost equal to

$$\mathcal{D}(A) = \{x \in {}^{\omega}2 \mid x \text{ has density 1 in } A\},$$

and therefore $\mathcal{D}$ selects a set from each measure class. It turns out that $\mathcal{D}(A)$ is $\boldsymbol{\Pi}^0_3$ — in fact it can be complete $\boldsymbol{\Pi}^0_3$. I will present some partial results on the complexity of $\mathcal{D}(A)$ for various As.

This is joint work with Riccardo Camerlo.

Alessandro Andretta
University of Torino (Italy)

Week 2, Monday June 22, 09:00–09:50

Indestructible strong compactness but not supercompactness

I will discuss the construction of a model containing a supercompact cardinal κ whose strong compactness, but not supercompactness, is fully indestructible under κ-directed closed forcing. This is joint work with Joel Hamkins and Grigor Sargsyan.

Arthur Apter
Baruch College and The Graduate Center, CUNY (USA)

Week 1, Tuesday June 16, 14:10–15:00

Rigid iterations with systems of side conditions

David Aspero
University of Barcelona (Spain)

Week 1, Tuesday June 16, 15:15–16:05

$C^{(n)}$-cardinals

Let $C^{(n)}$ be the proper class of cardinals κ that are Σ_n-correct in V, meaning that V_κ is a Σ_n-elementary substructure of V. We will consider several types of large cardinal notions obtained by requiring that both the critical point κ of an elementary embedding $j : V \to M$ and its image under j are in $C^{(n)}$. These are the $C^{(n)}$-cardinals. One of the results is that Vopěnka's Principle is equivalent to the existence of a $C^{(n)}$-extendible cardinal, for every n.

Joan Bagaria
University of Barcelona (Spain)

Week 2, Tuesday June 23, 09:00–09:50

Cardinal invariants of analytic quotients

Jörg Brendle
Kobe University (Japan)

Week 1, Tuesday June 16, 10:05–10:55

Standard universal dendrites as small Polish structures

Small Polish structures have been introduced recently by Krupiński to apply some model theoretic notions in a descriptive set theoretic context. Examples of small Polish structures will be presented, with a discussion of some open problems. The case of dendrites will be particularly examined.

Riccardo Camerlo
Polytechnic of Turin (Italy)

Week 2, Monday June 22, 15:15–16:05

On dimension and Borel reducibility

Borel reducibility of equivalence relations was introduced by Friedman and Stanley in 1989. This powerful concept allows us to use methods of descriptive set theory to compare the complexity of classification problems from other areas of mathematics.

Our starting point will be the amazing result, due to Hjorth and Thomas in 1998-2001, that the complexity of the classification problem for torsion-free abelian groups of finite rank increases strictly with the rank. Other invariants besides just the rank can be used. For instance, Thomas showed that even once the rank is fixed, the classification subproblems for p-local and q-local groups have incomparable complexities.

In each of these results, the "dimension" of the classification problem plays a crucial role. This leaves open the following natural question, which we will discuss in this talk: To what extent do the dimensions of two classification problems decide their relationship under Borel reducibility?

Samuel Coskey

Week 2, Thursday June 25, 14:10–15:00 *City University of New York (USA)*

Non-proper elementary embeddings beyond $L(V_{\lambda+1})$

So far the strongest great cardinals hypothesis that has received a deep and shared analysis is the existence of an elementary embedding j from $L(V_{\lambda+1})$ to itself, for some $\lambda > cp(j)$. There were various attempts to define hypotheses stronger than I0, but Woodin's approach caught my attention: since he found several similarities between $L(V_{\lambda+1})$ under I0 and $L(\mathbb{R})$ under AD, he continued to carry on the comparison trying to find a hypotesis similar to $\mathrm{AD}_{\mathbb{R}}$, constructing a sequence of $E^0_\alpha(V_{\lambda+1})$ such that $V_{\lambda+1} \subseteq E^0_\alpha(V_{\lambda+1}) \subseteq V_{\lambda+2}$, that imitates the construction of the minimum model of $\mathrm{AD}_{\mathbb{R}}$.

My attention is focused on the properties of the elementary embeddings from $L(E^0_\alpha(V_{\lambda+1}))$ to itself, and the first property that I analyzed is *properness*, i.e. the cofinality in $\Theta^{L(E^0_\alpha(V_{\lambda+1}))}$ of the fixed points of the embedding, that it turns out is quite important in preserving the similarity with determinacy. The first original result is the existence of an α and a $j : L(E^0_\alpha(V_{\lambda+1})) \prec L(E^0_\alpha(V_{\lambda+1}))$ that is not proper. This both validates the definition of proper elementary embedding, since it states for the first time that the definition is not trivial, and fills a gap in a Theorem by Woodin that is fundamental for this new research.

Vincenzo Dimonte

Week 1, Friday June 19, 17:05–17:30 *University of Turin (Italy)*

Tukey degrees of ultrafilters

This is joint work with Stevo Todorcevic. Let U and V be ultrafilters on countable base sets. We say that V is Tukey reducible to U ($V \leq_T U$) if there is a "Tukey map" $g : V \to U$, meaning that g maps unbounded subsets of V to unbounded subsets of U. Equivalently, there is a "cofinal" map $f : U \to V$ which maps cofinal subsets of U to cofinal subsets of V. Tukey reducibility on ultrafilters isa generalization of Rudin-Keisler reducibility. We present a canonization of cofinal maps from a p-point into another ultrafilter as monotone continuous functions, and some analogues for ultrafilters with similarities to p-points. We also give some results on the structure of the Tukey types of ultrafilters on ω and FIN, concentrating on p-points, selective ultrafilters, and ultrafilters with similar properties.

Natasha Dobrinen

Week 1, Thursday June 18, 17:05–17:30 *University of Denver (USA)*

Applications of Descriptive Set Theory in the Geometry of Banach spaces

We will review some recent advances on the interaction between Descriptive Set Theory and the Geometry of Banach spaces. We will concentrate around problems which can be traced back to the beginnings of Banach Space Theory and asking whether certain classes of separable Banach spaces admit universal spaces with special properties. Recently, all these problems are solved and the crucial conceptual vehicle for arriving to the solution is the notion of a strongly bounded class of separable Banach spaces. Beside its intrinsic functional-analytic interest, this notion points out towards a more general phenomenon which seems to be of interest also to set theorists.

Pandelis Dodos
National Technical University of Athens (Greece)

Week 2, Tutorial:
Monday June 22, 10:15–11:05
Tuesday June 23, 10:15–11:05
Wednesday June 24, 10:15–11:05

Nonseparable UHF algebras

Uniformly HyperFinite (UHF) algebras are those C^* algebras in which every finite subset is 'near' a finite-dimensional full matrix subalgebra. This can be formalized in three different ways, all three being equivalent in the separable case.

Separable UHF algebras were classified in the 1960s by Glimm and Dixmier. Dixmier asked whether three definitions are equivalent in the nonseparable case. I will give a complete answer to this question. Then I will state some even more basic questions that we could not answer. This is a joint work with Takeshi Katsura.

Ilijas Farah
York University, Toronto (Canada)

Week 1, Friday June 19, 08:50–09:40

Complexity of isomorphism between Banach spaces and inevitable list of Gowers

We shall discuss two directions of classification of separable Banach spaces up to isomorphism, and their interaction. The first direction is a classification "up to subspaces", as initiated by the work of Gowers: in this direction one looks for a list of "elementary" spaces, such that any Banach space contains a subspace isomorphic to one in the list. The second direction is a classification "by complexity", where each space is characterized by the complexity of the relation of isomorphism between its subspaces.

Valentin Ferenczi
University of Sao Paulo (Brazil)

Week 2, Thursday June 25, 09:00–09:50

Inner models for huge cardinals and Strong Chang's Conjecture

Chang's Conjectures are strengthenings of the Lowenheim-Skolem theorem. Given an arbitrary structure A they ask for an elementary substructure B where the cardinalities of the intersections of B with various predicates are specified in advance.

If A is well-founded one could also ask that the transitive collapse of B be large; i.e. that B have strong condensation properties. In this talk a strong Chang's Conjecture of this form is presented. The consistency strength of this Chang's Conjecture holding at ω_3 is between a 2-huge and a huge cardinal.

Matt Foreman
University of California, Irvine (USA)

Week 1, Thursday June 18, 08:50–09:40

Models for Measure Preserving Transformations

Measure preserving transformations arise in many different settings. Each setting gives its own topology on the collection of transformations and some provide algebraic structure as well.

A natural question is whether two different settings have the same generic dynamical properties and give the same Borel structure on the measure preserving transformations.

Dan Rudolph gave a meta-conjecture that all settings are equivalent. In these two talks we make this precise in various ways and prove it. We also introduce some new settings such as the space of rational invariant measures.

Matt Foreman

Week 2, Friday June 26, 09:00–09:50 *University of California, Irvine (USA)*

Fodor-type reflection principle and its "mathematical" characterizations

Fodor-type Reflection Principle (FRP) is the assertion that the following FRP(κ) holds for all regular cardinals $\kappa > \aleph_1$:

FRP(κ): For any stationary $E \subseteq E^\kappa_\omega$ and $g : E \to [\kappa]^{\aleph_0}$, there is an $I \in [\kappa]^{\aleph_1}$ such that

(1) $cf(I) = \omega_1$;
(2) I is closed with respect to g; and
(3) for any $f : E \cap I \to \kappa$ if $f(\alpha) \in g(\alpha) \cap \alpha$ for all $\alpha \in E \cap I$, then there is a $\beta^* \in I$ such that $f^{-1}``\{\beta^*\}$ is stationary in $\sup(I)$.

Using a new characterization of FRP we show that many reflection theorems originally obtained as consequences of Axiom R are actually equivalent to FRP over ZFC. The following two are among such assertions equivalent to FRP:

- For every locally countably compact topological space X, if all subspaces of X of cardinality $\leq \aleph_1$ are metrizable, then X itself is metrizable.
- For any graph G, if all subgraphs of G of cardinality $\leq \aleph_1$ have countable coloring number, then G itself has countable coloring number.

The main results of this talk are obtained in a joint research with Lajos Soukup, Hiroshi Sakai and Toshimichi Usuba.

Sakaé Fuchino

Week 1, Monday June 15, 15:15–16:05 *Chubu University (Japan)*

The Descriptive Complexity of Free Bernoulli Subflows

I will talk about some joint work with Jackson and Seward on some applications of descriptive set theory to problems in dynamical systems. We have previously shown that free Bernoulli subflows exist for all countably infinite groups. In this talk I will focus on two problems: one is to characterize the descriptive complexity of all (minimal) free Bernoulli subflows, and another is to determine the exact complexity of the isomorphism relation of (minimal) free Beroulli subflows. For the first problem the complete answer is given, which also introduces a new concept that seems to have never been studied in combinatorial group theory. For the second problem the complete answer is not known, but I will talk about partial results obtained.

Su Gao

Week 2, Thursday June 25, 10:15–11:05 *University of North Texas (USA)*

Ramsey-like cardinals

One of the less known characterizations of a Ramsey cardinal κ involves the existence of certain types of elementary embeddings for transitive sets of size κ satisfying a large fragment of ZFC. I introduce new large cardinal axioms by isolating and generalizing the key properties of Ramsey embeddings and show that they form a natural hierarchy between weakly compact cardinals and measurable cardinals. The stronger of these large cardinal notions are better suited than Ramsey cardinals for indestructiblity arguments. The weaker of the new large cardinals further our knowledge about the elementary embedding properties of smaller large cardinals, in particular those still consistent with $V = L$. A large portion of this work is joint with Philip Welch.

Victoria Gitman
Week 1, Friday June 19, 11:45–12:10 *New York City College of Technology (USA)*

General relations of the set-theoretic universe to its forcing extensions and grounds

I shall describe recent work focussed on general relations of the set-theoretic universe to its forcing extensions and grounds. A set theoretical assertion is forceable (or possible) if it holds in some forcing extension, and necessary if it holds in all forcing extensions.

These concepts are fundamentally modal in nature, and it is natural to inquire which modal assertions are valid for this forcing interpretation. What is the modal logic of forcing? The answer, established in joint work with B. Loewe, is that if ZFC is consistent, then the ZFC-provably valid principles of forcing are exactly those in the modal theory known as S4.2.

The ideas admit a duality, looking downward to ground models rather than upward to forcing extensions, and in this case we have established the same S4.2 theory of validities, provided that ground models are downward directed. The Downward Directedness hypothesis is the principal open question of set-theoretic geology, introduced by myself, Fuchs and Reitz, and one of our initial results is that every model of ZFC is the Mantle—the intersection of all grounds—of another model of ZFC. Some of this analysis engages pleasantly with various philosophical views on the nature of mathematical existence.

Joel David Hamkins
Week 1, Wednesday June 17, 08:50–09:40 *City University of New York (USA)*

The descriptive set theory of unitary group representations

Greg Hjorth
Week 2, Friday June 26, 16:30–17:20 *University of Melbourne (Australia)*

Large cardinals and the continuum function

By Easton's results, it is known that ZFC can prove very little about the continuum function. However, this result does not carry over directly to extensions of ZFC by large cardinals. This is due to the reflection properties of large cardinals, such as a measurable cardinal. We study the following problem using a cardinal-preserving forcing: given a universe of sets with large cardinals, which continuum functions are compatible with given large cardinals? This is a generalization of Easton's theorem to situations with large cardinals. As a bonus, we can extend these results naturally to include singular cardinals which will fail SCH, studying the compatibility of continuum functions and cardinals failing SCH.

Radek Honzik
Week 1, Monday June 15, 11:45–12:10 *Charles University, Prague (Czech Republic)*

Katetov order on Borel ideals

We will present two dichotomies for Borel ideals involving the Katetov order.

Michael Hrušák
Week 2, Monday June 22, 16:30–17:20 *University of Mexico (Mexico)*

Real Blackwell Determinacy

Blackwell games are infinite games with (slightly) imperfect information while Gale-Stewart games are infinite games with perfect information. Martin proved that the Axiom of Determinacy (AD) implies the Axiom of Blackwell Determinacy (Bl-AD) and conjectured the converse. We show that the Axiom of Real Blackwell Determinacy (Bl-$AD_{\mathbb{R}}$) implies the consistency of AD and hence the consistency of Bl-$AD_{\mathbb{R}}$ is strictly greater than that of AD. This is a joint work with David de Kloet and Benedikt Löwe.

Daisuke Ikegami
Week 1, Tuesday June 16, 11:10–11:35 *University of Amsterdam (Netherlands)*

A precipitous club guessing ideal on ω_1

For each club guessing sequence, we can define a natural ideal associated with it, called a club guessing ideal. I will talk about how to prove the consistency relative to a measurable cardinal that there is a precipitous club guessing ideal on ω_1 but the non-stationary ideal on ω_1 is not precipitous. If time permits, I will discuss other results about the precipitousness of natural ideals.

Tetsuya Ishiu
Week 1, Thursday June 18, 10:05–10:55 *Miami University (USA)*

New partition results from AD

Stephen Jackson
Week 1, Wednesday June 17, 10:00–10:50 *University of North Texas (USA)*

Substituting Supercompactness by Strong Unfoldability

Strongly unfoldable cardinals are relatively low in the hierarchy of large cardinals, they lie well below measurable cardinals and are consistent with $V = L$. In this talk I will discuss recent results that have shown how strong unfoldability can serve as a highly efficient substitute for supercompactness in several large cardinal phenomena. In particular, I will discuss a Laver-like indestructibility theorem for strong unfoldability and a Baumgartner-like relative consistency proof of a fragment PFA: If κ is a strongly unfoldable cardinal, then there is a model in which kappa is indestructible by all $<\kappa$-closed, κ^+-preserving forcing notions; and there is a model in which PFA holds for forcing notions that preserve either $\aleph_2$ or $\aleph_3$.

This is joint work with Joel David Hamkins.

Thomas Johnstone
Week 1, Monday June 15, 16:30–16:55 *New York City College of Technology (USA)*

On definability of some counterexamples in descriptive set theory

It is known since early studies on constructibility and forcing that counterexamples to some classical theorems of descriptive set theory consistently exist at suitable projective levels.

This includes, e.g.,

(1) a non-measurable Δ^1_2 set
(2) a nonconstructible Δ^1_3 real
(3) sets that witness failure of Separation for Π^1_3,

and many more.

This naturally led to a question whether counterexamples consistently exist at n-th level of the hierarchy under the assumption that they do not exist at levels below n. Some results in this direction, for arbitrary n, related to definable nonconstructible reals, prewellorderings, Separation, Reduction, are known since mid-1970s, mainly to Leo Harrington, but remain unpublished. The main goal of the talk will be to present proofs of these theorems up to major details, and explain related difficulties.

No general results like this are known so far for measurability and other regularity properties. Although the steps from $n = 2$ to $n = 3$ and from $n = 3$ to $n = 4$ have been resolved by methods that do not generalize to higher levels.

Some other open problems will be discussed.

Vladimir Kanovei
Institute for Information Transmission Problems,
Moscow (Russia)

Week 2, Tuesday June 23, 15:15–16:05

Different ways to produce non-special ω_2-Aronszajntrees

The consistency strength of “no ω_2-Suslin trees” is still open, but the consistency strength of “no non-special ω_2-Aronszajn trees” is known to be a weakly compact. We discuss some statements in between those two and give some results about special and non-special ω_2-Aronszajn trees respectively. We might also mention some related results if time allows.

Bernhard Koenig
University of Toronto (Canada)

Week 1, Friday June 19, 11:10–11:35

The Weak Reflection Principle Versus the Reflection Principle

The *Weak Reflection Principle* for ω_2 is the statement that for every stationary set $S \subseteq P_{\omega_1}(\omega_2)$, there is an uncountable ordinal α in ω_2 such that $S \cap P_{\omega_1}(\alpha)$ is stationary in $P_{\omega_1}(\alpha)$. The *Reflection Principle* for ω_2 is the statement that for every stationary set $S \subseteq P_{\omega_1}(\omega_2)$, there is an ordinal α in ω_2 with cofinality ω_1 such that $S \cap P_{\omega_1}(\alpha)$ is stationary in $P_{\omega_1}(\alpha)$. A long outstanding problem in set theory has been the question whether the Weak Reflection Principle for ω_2 implies the Reflection Principle for ω_2. In this talk I will discuss my recent solution to this problem.

John Krueger
University of California, Berkeley (USA)

Week 1, Monday June 15, 14:10–15:00

Partitions and Indivisibility Properties of Countable Dimensional Vector Spaces

(Joint work with Lionel Nguyen Van The, Norbert Sauer)

We investigate infinite versions of vector and affine space partition results, and thus obtain examples and a counterexample for a partition problem for relational structures. In particular we provide two examples of an age indivisible relational structure which is not weakly indivisible.

Claude Laflamme
University of Calgary (Canada)

Week 2, Thursday June 25, 15:15–16:05

Universally measurable sets in generic extensions

Paul Larson
Miami University (USA)

Week 1, Tuesday June 16, 08:50–09:40

Generic constructions of Banach spaces

The aim of this talk is to present a forcing construction “à la Cohen” of generic Banach spaces. These spaces are Gurarij spaces, and in the case of the non-separable context, they can be non-isomorphic. These constructions can also be used to distinguish the existence of different kind of uncountable biorthogonal-like sequences. This is a joint work with S. Todorčević.

Jordi Lopez-Abad
Instituto de Ciencias Matemáticas (CSIC), Madrid (Spain)

Week 2, Monday June 22, 11:20–12:10

Eventually Different Forcing at the Second Level of the Projective Hierarchy

There is a general connection between notions of forcing adding real numbers and notions of measurability on the real line. Using general results by Ikegami on the relationship between the measurability of $Delta^1_2$ and $Sigma^1_2$ sets and the existence of quasi-generics over models of the type $L[x]$, we characterize the statements “every $Delta^1_2$ set has the Baire property in the eventually different topology” and “every $Sigma^1_2$ set has the Baire property in the eventually different topology”. This is joint work with Jörg Brendle (Kobe).

Benedikt Löwe
University of Amsterdam (Netherlands)

Week 1, Wednesday June 17, 11:00–11:50

Cofinal types of definable directed orders

We discuss some recent developments in the theory of cofinal similarity types of definable directed orders. We detail the relevant techniques: definable Tukey maps, perfect set theorems and Ramsey methods.

Tamás Mátrai
University of Toronto (Canada)

Week 2, Tuesday June 23, 16:30–17:20

Metric structures and applications to the theory of topological groups

It has long been known that the automorphism group of a countable first-order structure is (isomorphic to) a closed subgroup of S_∞, and conversely any such group is(isomorphic to) the automorphism group of some countable first order structure (which can also assumed to be relational and ultrahomogeneous). This has led to an interesting interplay between model theory and the descriptive theory of actions of subgroups of S_∞. In this talk, we will explain how the concept of metric structure (as introduced by Ben Yaacov, Berenstein, Henson and Usvyastov) leads to a similar interplay between so-called continuous logic and the descriptive theory of Polish groups and their actions.

We will in particular explain why this leads to an extension of the concept of ample generics (introduced by Kechris and Rosendal) and present examples and applications of this new concept. Finally, if time allows, we will discuss some related open questions.

Julien Melleray
University of Lyon (France)

Week 2, Tuesday June 23, 11:20–12:10

Proper translation

Heike Mildenberger
Week 1, Thursday June 18, 14:10–15:00 *KGRC, University of Vienna (Austria)*

Forceless, ineffective, powerless proofs of descriptive set-theoretic dichotomy theorems.

Since its inception, the study of definable subsets of the real numbers has been dominated by a variety of structural dichotomy theorems. In recent times, the proofs of these theorems have grown increasingly complex and dependent upon techniques from mathematical logic. We will discuss an approach to giving classical proofs of these results which is motivated by ideas from graph theory.

Ben Miller
Week 2, Friday June 26, 10:15–11:05

Towers in Boolean algebras

A *tower* in a BA A is a strictly increasing sequence of regular length of elements of A, with sum 1. $\mathfrak{t}_{\mathrm{spect}}(A) = \{|X| : X \text{ is a tower in } A\}$, and $\mathfrak{t}(A) = \min \mathfrak{t}_{\mathrm{spect}}(A)$. Note that $\mathfrak{t}(A)$ is not defined for every BA.

We survey what is known about these functions in arbitrary BAs. Partly these results concern other cardinal functions generalized from the continuum cardinal case: $\mathfrak{p}(A) = \min\{|X| : \sum X = 1$ and $\sum F \neq 1$ for every finite $F \subseteq X\}$; $\mathfrak{a}(A) = \min\{|X| : X$ is a partition of unity in $A\}$; $\mathfrak{s}(A) = \min\{|X| : X$ splits $A\}$ (which means that for every nonzero $a \in A$ there is an $x \in X$ such that $a \cdot x \neq 0 \neq a \cdot -x$).

Results whose proofs are sketched:

(1) There is an atomless BA A such that $\mathfrak{p}(A) < \mathfrak{t}(A)$.
(2) For $\aleph_0 < \kappa < \lambda$ regular, there is an atomless BA A such that $\mathfrak{s}(A) = \mathfrak{t}(A) < \mathfrak{a}(A)$.
(3) There is an atomless interval algebra A such that $\mathfrak{a}(A) < \mathfrak{t}(A)$
(4) If M is a nonempty set of regular cardinals, then there is an atomless BA A such that $\mathfrak{t}_{\mathrm{spect}}(A) = M$.
(5) We give a characterization of those linear orders whose interval algebras have towers.
(6) A similar characterization is known for pseudo-tree algebras, and we sketch the special case of tree algebras.

J. Donald Monk
Week 1, Friday June 19, 10:05–10:55 *University of Colorado, Boulder (USA)*

A rigid iteration with side conditions

Using the technique of Rigid Iterations with Side Conditions we show that some consequences of BPFA are consistent with the continuum large. This is a joint work with David Asperó.

Miguel Angel Mota
Week 1, Tuesday June 16, 16:30–16:55 *University of Barcelona (Spain)*

A universality property for analytic equivalence relations and quasi-orders

Some years ago, Louveau and Rosendal showed that the relation of bi-embeddability for countable graphs is complete for analytic equivalence relations. This is in strong contrast to the case of the isomorphism relation, which as an equivalence relation on graphs (or on any class of countable structures consisting of the models of an $L_{\omega_1\omega}$-sentence) is far from complete. In this talk I will present a strenghtening of the Louveau-Rosendal result by showing that not only does bi-embeddability give rise to analytic equivalence relations which are complete under Borel reducibility, but in fact any analytic equivalence relation is Borel equivalent to such a relation. This proves that, quite surprisingly, the logic relation of bi-embeddability is able to capture the great complexity of the whole structure of analytic equivalence relations (up to Borel-equivalence). Similar results can also be obtained looking at various notions of morphism naturally arising e.g. in Model Theory and Descriptive Set Theory. Moreover, the techniques introduced allow to answer to some questions posed by Louveau and Rosendal about the possible relationships between isomorphism and bi-embeddability.

This is partially joint work with Sy-David Friedman and Riccardo Camerlo.

Luca Motto Ros
KGRC, University of Vienna (Austria)

Week 2, Tuesday June 23, 14:10–15:00

Structural Ramsey theory and topological dynamics

In 2003, Kechris, Pestov and Todorcevic showed that some dynamical properties of closed subgroups of the permutation group of the naturals are closely related to Ramsey-type properties of certain classes of finite structures. This series of talks will present different aspects of this connection via various examples and will focus on the following topics:

- Closed subgroups of S_∞ and countable ultrahomogeneous structures;
- Finite structural Ramsey theory, extreme amenability and universal minimal flows;
- Infinite structural Ramsey theory and oscillation stability.

Lionel Nguyen Van The
University of Neuchatel (Switzerland)

Week 2, Tutorial:
Wednesday June 24, 11:20-12:10
Thursday June 25, 11:20-12:10
Friday June 26, 11:20-12:10

Playing with countable support iteration

Countable support iteration of definable forcings is often used in Set Theory of the Reals. I'll discuss a game of length ω_1 which seems to be responsible for many combinatorial properties of the generic extension. This is connected to the Parametrized Diamond Principles and developes the Covering Property Axiom.

Janusz Pawlikowski
University of Wroclaw (Poland)

Week 1, Thursday June 18, 15:15–16:05

Diamond on successor of singulars

We discuss the work of Shelah, Zeman, Gitik and the speaker concerning diamond on successors of singulars. A few surprising applications would be indicated as well.

Assaf Rinot
Tel-Aviv University (Israel)

Week 1, Thursday June 18, 11:45–12:10

Infinite asymptotic games and an exact Ramsey principle for block sequences

We prove an exact, i.e., formulated without Δ-expansions, Ramsey principle for infinite block sequences in vector spaces over countable fields, where the two sides of the dichotomic principle are represented by respectively winning strategies in Gowers' block sequence game and winning strategies in the infinite asymptotic game. This allows us to recover Gowers' dichotomy theorem for block sequences in normed vector spaces by a simple application of the basic determinacy theorem for infinite asymptotic games.

Christian Rosendal
University of Illinois, Chicago (USA)

Week 2, Friday June 26, 15:15–16:05

On forcing with σ-ideals of closed sets

I would like to talk about idealized forcing for σ-ideals generated by closed sets. I will speak about iterations of such forcings and descriptive set-theoretic characterization of conditions in the iteration.

Marcin Sabok
University of Wroclaw (Poland)

Week 1, Thursday June 18, 16:30–16:55

On partitions of relational structures

Partition properties of homogeneous relational structures are determined by their automorphism groups. Hence permutation groups closed in the finitary topology have partition properties, lifted from the partiton properties of the underlying homogeneous structures. Those properties can be defined using permutation group notions only. On the other hand due to results of Kechris, Pestov and Todorcevic and Pestov various connections between actions of topological groups on compacta and relational Ramsey theory have become known.

In order to illustrate various of those partition properties and their interrelationships the simpler case of point partitions will be discussed.

Norbert Sauer
University of Calgary (Canada)

Week 2, Wednesday June 24, 09:00–09:50

Bounded forcing axioms and reflection

We study mouse reflection in the presence of BPFA, the bounded proper forcing axiom. It turns out that under Woodin's $P_{\max}$ axiom (*), BPFA is equivalent to BMM^{++}, a strong version of bounded Martin's maximum. We also present new information about (†), the statement according to which every stationary set preserving forcing is semiproper.

Ralf Schindler
Universität Muenster (Germany)

Week 1, Monday June 15, 10:05–10:55

Thin equivalence relations in scaled pointclasses

We give an inner model-theoretic proof that every thin $\mathbf{\Sigma}_1^{J_\alpha(\mathbb{R})}$ equivalence relation is $\mathbf{\Pi}_1^{J_\alpha(\mathbb{R})}$ if α begins a gap and $J_\alpha(\mathbb{R})$ is admissible, assuming $\mathrm{AD}^{J_\alpha(\mathbb{R})}$. This is joint work with Ralf Schindler.

Philipp Schlicht
Universität Bonn (Germany)

Week 1, Friday June 19, 16:30–16:55

Homogeneously Suslin sets in mice with Woodin cardinals

In M_n, the minimal fine-structural iterable proper class model with n Woodin cardinals, all homogeneously Suslin sets are $\mathbf{\Delta^1_{n+1}}$. This talk will describe the main ideas involved in the proof of this fact for $n = 1$.

Farmer Schlutzenberg

Week 1, Monday June 15, 11:10–11:35

Exploring Singular Cardinal Combinatorics

Dima Sinapova
Week 1, Tuesday June 16, 11:45–12:10 *University of California, Irvine (USA)*

On properties of families of sets

Lajos Soukup
Week 1, Friday June 19, 14:10–15:00 *Hungarian Academy of Sciences*

The Largest Large Cardinal and the Inner Model Hypothesis

Sy Friedman's *Inner Model Hypothesis* (IMH) asserts that every first-order parameter-free sentence in the language of arithmetic that holds in some outer model of V already holds in some definable inner model. In collaboration with Philip Welch and Hugh Woodin, he has shown that the IMH is consistent from a Woodin cardinal with an inaccessible above and has consistency strength at least that of measurable cardinals of arbitrarily high Mitchell order.

The IMH itself is incompatible with large cardinals and implies that the universe is minimal: By a theorem of Beller and Jensen, there exists a real x in any model of the IMH such that $L_\alpha[x]$ does not satisfy ZFC, for all α. For this same reason the IMH cannot be extended to sentences with arbitrary real parameters. (Consider "$\omega_1^{L[x]}$ is countable".)

We consider a variant of the IMH that is compatible with large cardinals, allows real parameters, and does not imply that the universe is minimal. Indeed, if the universe is sufficiently non-minimal, then this variant has a first-order formulation.

Mack Stanley
Week 1, Friday June 19, 15:15–16:05 *San Jose State University (USA)*

Some Consequences of Martin's Conjecture

In this talk, I will explore some of the consequences of Martin's Conjecture on degree invariant Borel maps. These include the strongest conceivable ergodicity result for the Turing equivalence relation, as well as the statement that the complexity of a universal countable Borel equivalence relation always concentrates on a null set.

Simon Thomas
Week 2, Friday June 26, 14:10–15:00 *Rutgers University (USA)*

Nonexistence of universal models at the successors of singular strong limit cardinals

In joint work with S.D. Friedman, we use interesting properties of some forcings which produce models for a failure of the SCH to show that these models do not have universal graphs at the successors of such singulars.

Katherine Thompson
Week 1, Monday June 15, 17:05–17:30 *KGRC, University of Vienna (Austria)*

The lifting problem for the group of measure preserving automorphisms of the unit interval

Asger Törnquist
Week 2, Thursday June 25, 16:30–17:20 *KGRC, University of Vienna (Austria)*

Rado's Conjecture, Saturation of the nonstationary ideal on ω_1, and two cardinal diamonds

We prove that Rado's Conjecture together with the saturation of the nonstationary ideal on ω_1 imply $\Diamond_{\omega_2}$ concentrated on ordinals of uncountable cofinality.

Victor Torres
Week 1, Tuesday June 16, 17:05–17:30 *University of Paris 7 (France)*

Some questions on the models of MM

We will briefly discuss some questions concerning the saturation properties of models of strong forcing axioms (MM, PFA, etc). Caicedo and Velickovic conjectured the following:

Assume MM and let W be an inner model with the same cardinals of the universe V. Then all sequences of ordinals of length $\aleph_1$ are already elements of the inner model W.

We shall discuss the motivations and the ground for the above conjecture as well as some partial results relating the conjecture to certain Ramsey properties and to Shelah's pcf theory.

Matteo Viale
Week 1, Monday June 15, 09:15–09:40 *University of Torino (Italy)*

CCC without random reals

We have known many proper (non-ccc) forcing notions which do not add random reals. We note that a sigma-centered forcing notion is a ccc forcing which does not add random reals.

We introduce two properties of forcing notions, and show that a forcing notion with one of these properties does not add random reals.

This is an extension of Zapletal's paper: Keeping additivity of the null ideal small. Proc. Amer. Math. Soc. 125 (1997), no. 8, 2443–2451.

Teruyuki Yorioka
Week 1, Thursday June 18, 11:10–11:35 *Shizuoka University (Japan)*

Games and sigma-porosity

Miroslav Zeleny
Week 2, Monday June 22, 14:10–15:00 *Charles University, Prague (Czech Republic)*