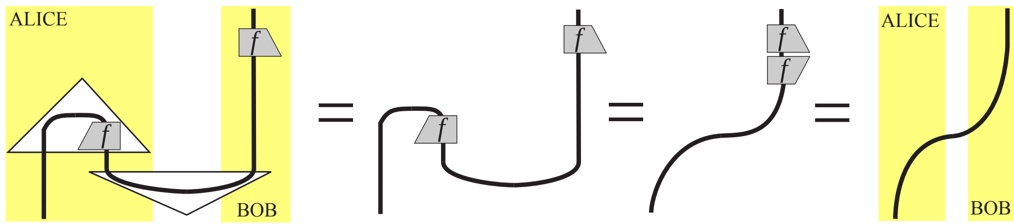
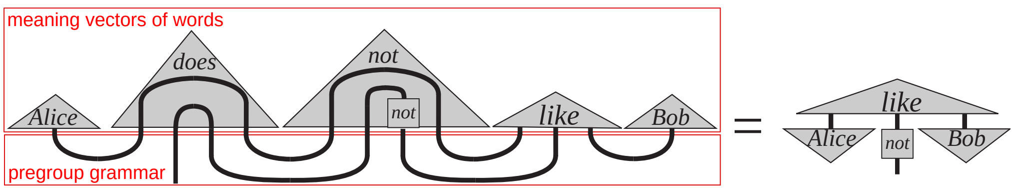
# Physics, Language, Maths & Music

(partly in arXiv:1204.3458)

Bob Coecke, Oxford, CS-Quantum

SyFest, Vienna, July 2013

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Sixth Framework Programme
Foundational Questions Institute
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# . . . via (some sort of) Logic

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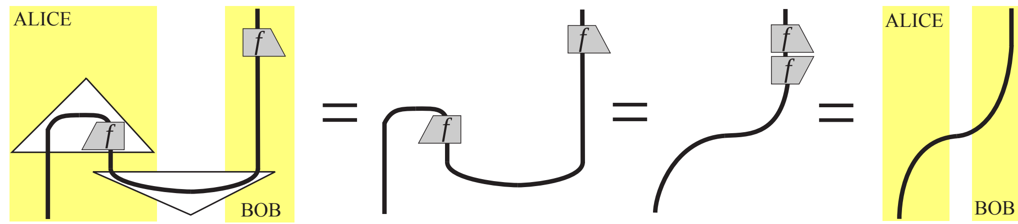


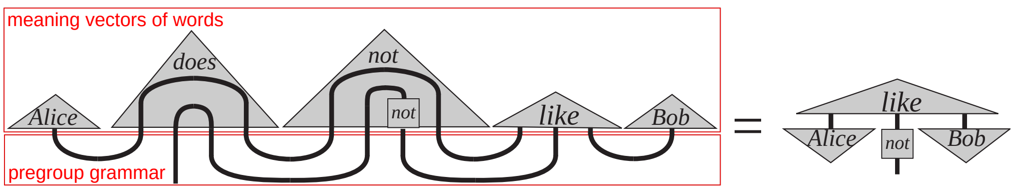


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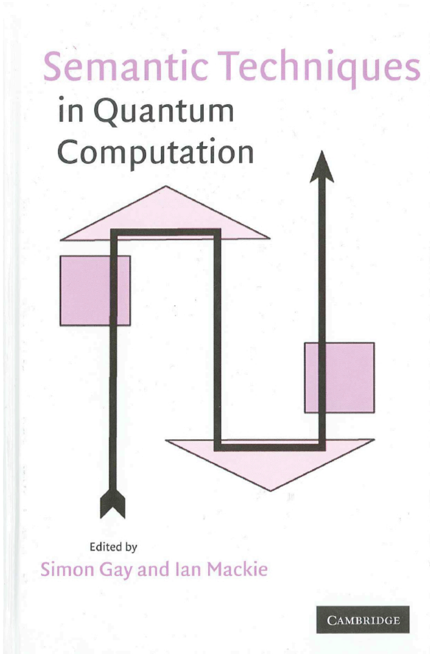
# — PHYSICS —





Samson Abramsky & BC (2004) *A categorical semantics for quantum protocols*. In: IEEE-LiCS'04. `quant-ph/0402130`

BC (2005) *Kindergarten quantum mechanics*. `quant-ph/0510032`

## — *genesis* —

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## — *the mathematics of it* —

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**WHY?**

## — *the mathematics of it* —

**Hilbert space stuff**: continuum, field structure of complex numbers, vector space over it, inner-product, etc.

**WHY?**

**von Neumann:** only used *it* since *it* was ‘available’.

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**von Neumann** crafted Birkhoff-von Neumann Quantum 'Logic' to capture the concept of **superposition**.

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**Quantum Computer Scientists:** Schrödinger is right!

## — *the game plan* —

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**Task 1.** Investigate which assumptions (i.e. which structure) on $\otimes$ is needed to deduce **physical phenomena**.

**Task 2.** Investigate wether such an "interaction structure" appear elsewhere in **"our classical reality"**.

— *wire and box language* —

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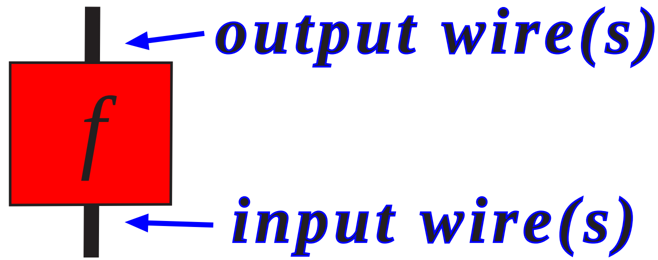
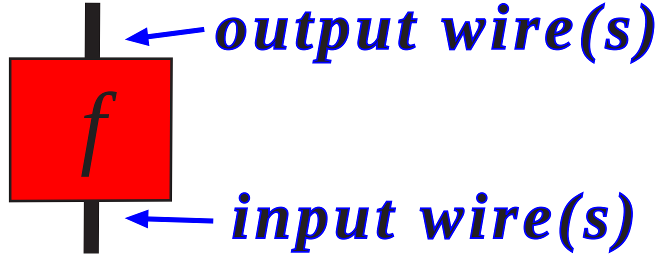
**Box :=** 

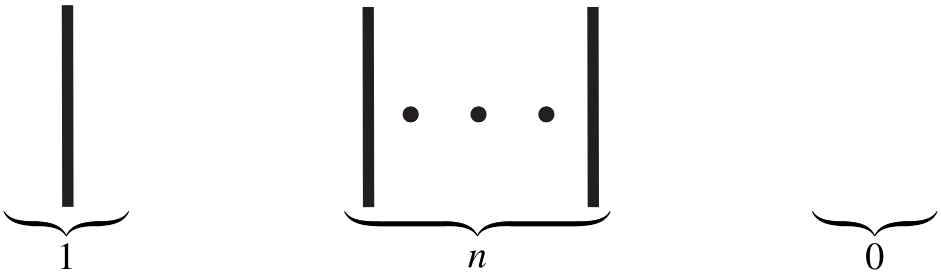

***Interpretation:*** wire := system ; box := **process**

— *wire and box language* —

**Box :=** 

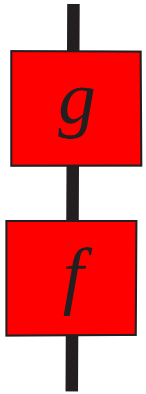

***Interpretation:*** wire := system ; box := **process**

one system: *n sub*systems: no system:

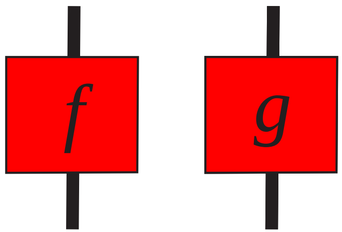


## — *wire and box games* —

*sequential* or *causal* or *connected* composition:

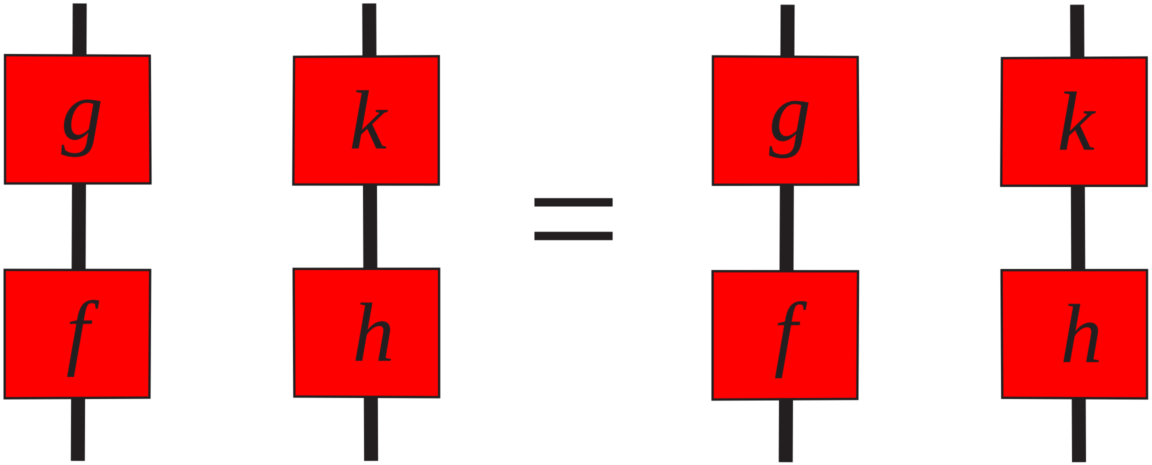
$g \circ f \equiv$ 


*parallel* or *acausal* or *disconnected* composition:

$f \otimes g \equiv$ 

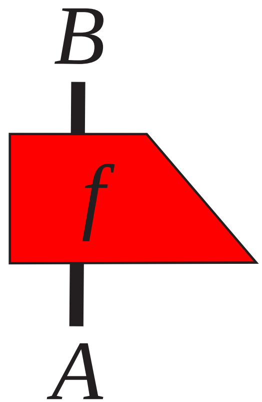
## — *merely a new notation?* —

$$(g \circ f) \otimes (k \circ h) = (g \otimes k) \circ (f \otimes h)$$



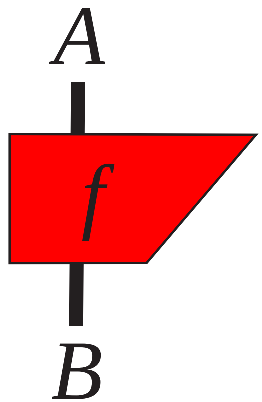
## — *quantitative metric* —

$$f : A \longrightarrow B$$

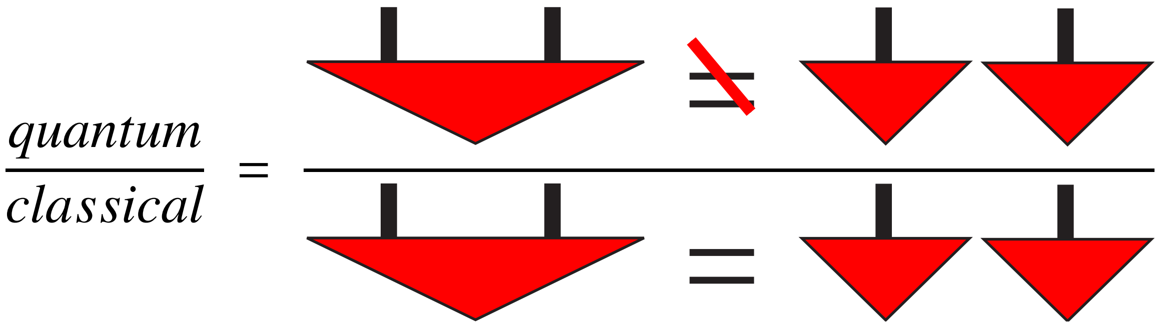


— ***quantitative metric*** —

$$f^{\dagger} : B \to A$$

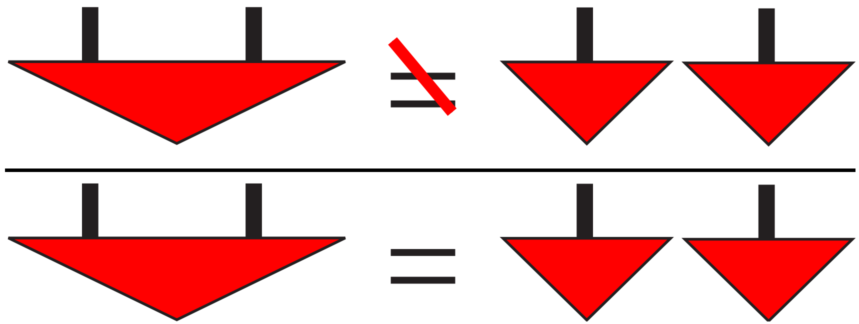


## — *asserting (pure) entanglement* —

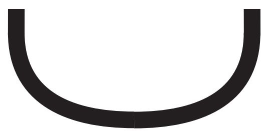


## — *asserting (pure) entanglement* —

$$\frac{\textit{quantum}}{\textit{classical}} =$$



⇒ **introduce ‘parallel wire’ between systems:**

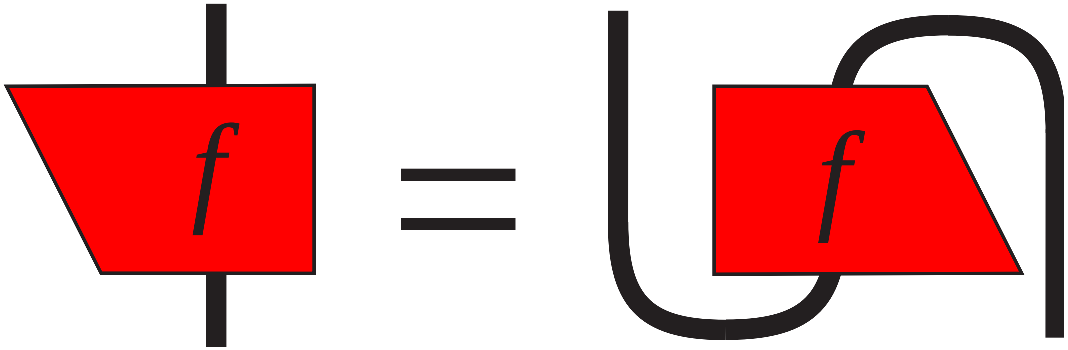


**subject to: only topology matters!**

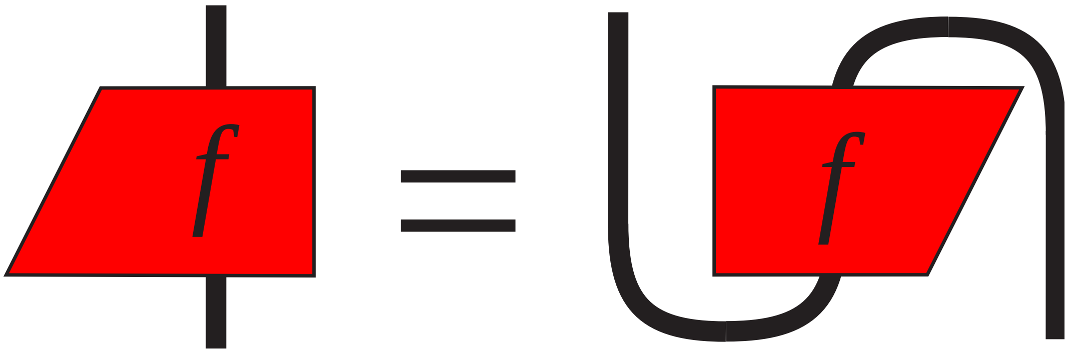
— *quantum-like* —

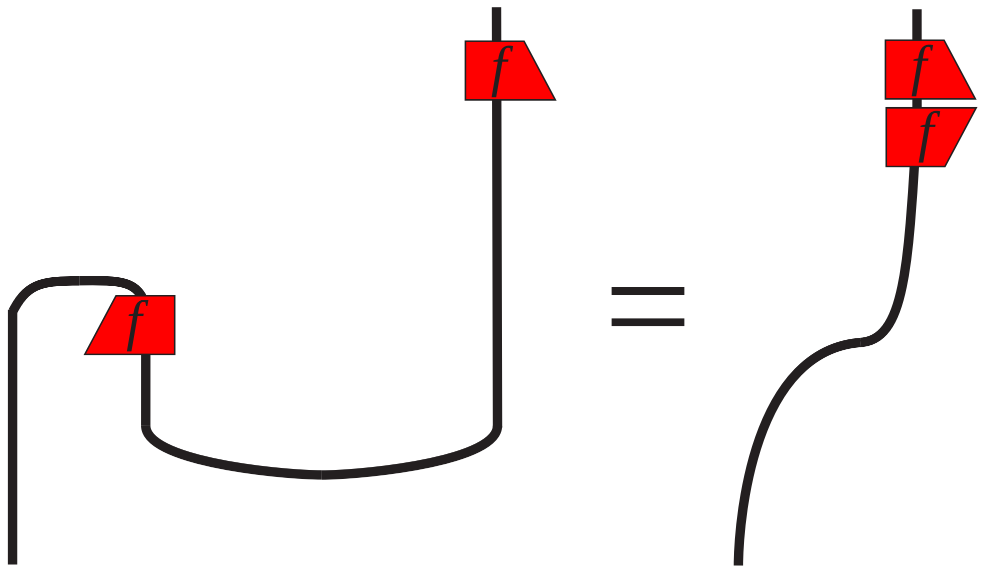
E.g.

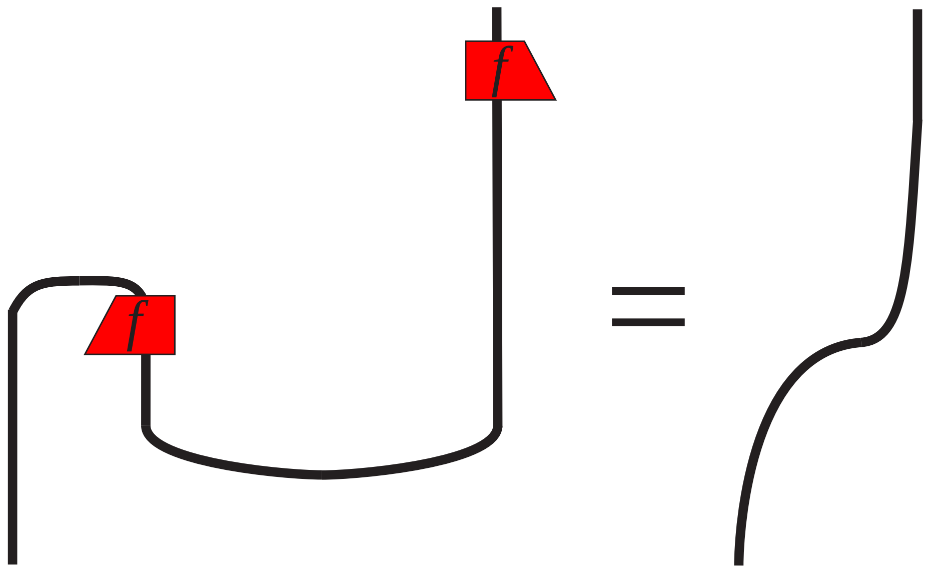
Transpose:




Conjugate:




f
f
=
f
f


f
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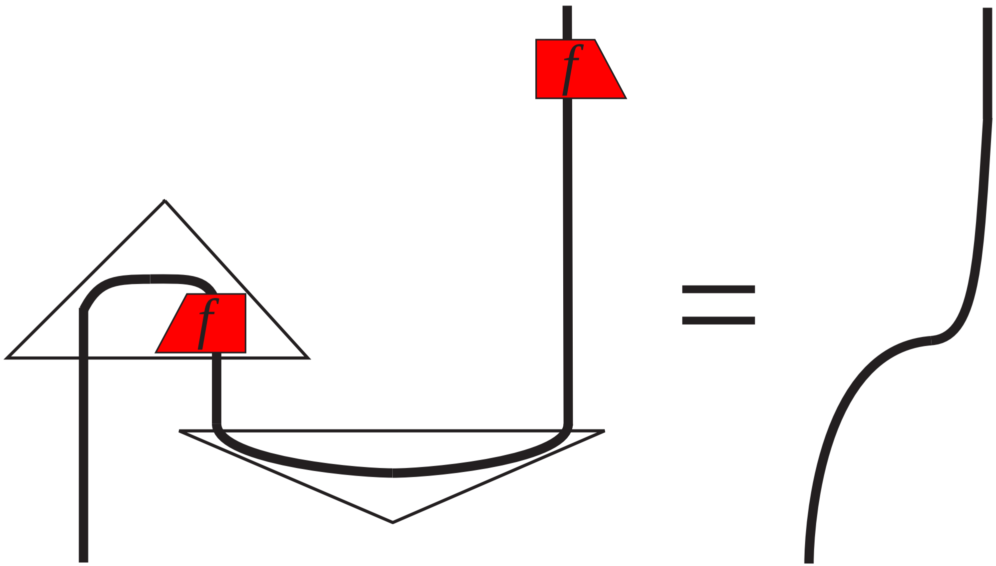
f
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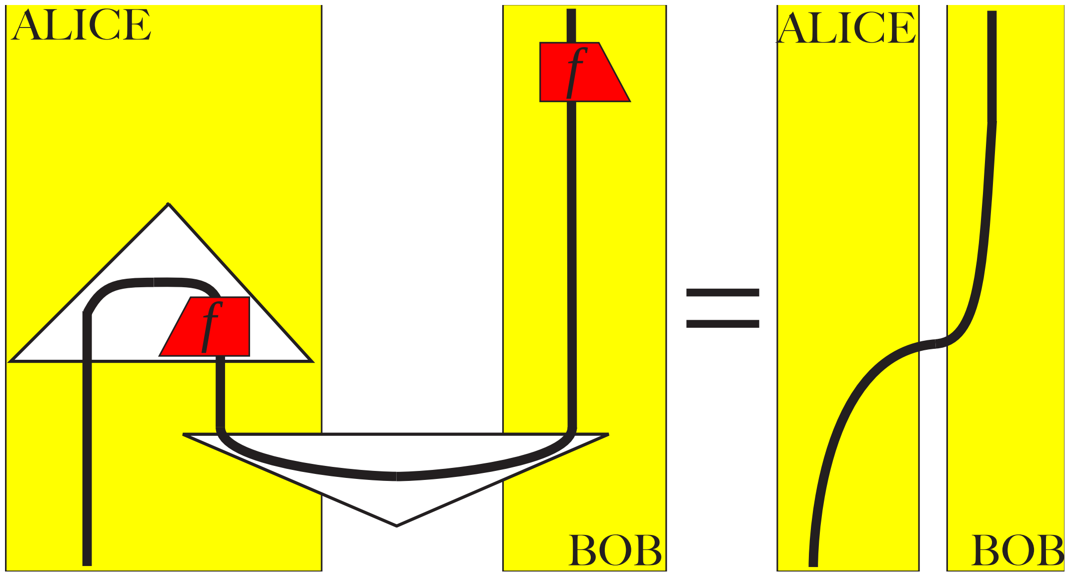


**⇒ quantum teleportation**

## — *symbolically: dagger compact categories* —

**Thm. [Kelly-Laplaza '80; Selinger '05]** *An equational statement between expressions in dagger compact categorical language holds if and only if it is derivable in the graphical notation via homotopy*.

**Thm. [Hasegawa-Hofmann-Plotkin; Selinger '08]** *An equational statement between expressions in dagger compact categorical language holds if and only if it is derivable in the dagger compact category of finite dimensional Hilbert spaces, linear maps, tensor product and adjoints*.

# — LANGUAGE—


BC, Mehrnoosh Sadrzadeh & Stephen Clark (2010) *Mathematical foundations for a compositional distributional model of meaning*. `arXiv:1003.4394`

— *the logic of it* —

# WHAT IS “LOGIC”?

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**Pragmatic option 1: Logic is structure in language.**

## — *the logic of it* —

# WHAT IS “LOGIC”?

## Pragmatic option 1: Logic is structure in language.

*“Alice and Bob ate everything or nothing, then got sick.”*

connectives $(\wedge, \vee)$ : *and, or*

negation $(\neg)$ : *not (cf. nothing = not something)*

entailment $(\Rightarrow)$ : *then*

quantifiers $(\forall, \exists)$ : *every(thing), some(thing)*

constants $(a, b)$ : *thing*

variable $(x)$ : *Alice, Bob*

predicates $(P(x), R(x, y))$ : *eating, getting sick*

truth valuation $(0, 1)$ : ***true, false***

$$(\forall z : Eat(a, z) \wedge Eat(b, z)) \wedge \neg(\exists z : Eat(a, z) \wedge Eat(b, z)) \Rightarrow Sick(a), Sick(b)$$

# WHAT IS "LOGIC"?

**Pragmatic option 1: Logic is structure in language.**

**Pragmatic option 2: Logic lets machines reason.**

— *the logic of it* —

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E.g. automated theory exploration, ...

## WHAT IS "LOGIC"?

**Pragmatic option 1: Logic is structure in language.**

**Pragmatic option 2: Logic lets machines reason.**

**Our framework appeals to both senses of logic, and moreover induces important new applications:**

From truth to meaning in natural language processing:

QUANTUM LINGUISTICS Leap forward for artificial intelligence

— NewScientist (December 2010)

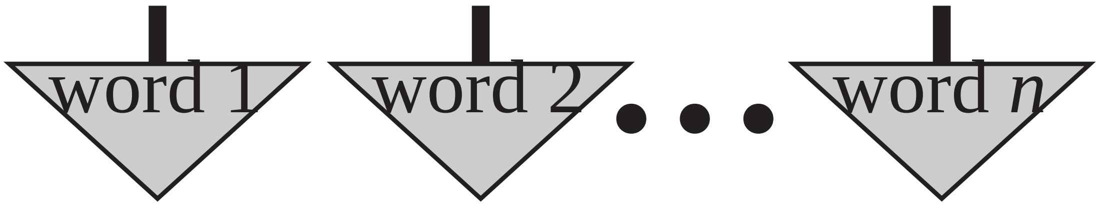
WEEKLY 11 December 2010

Automated theorem generation for graphical theories:

— http://sites.google.com/site/quantomatic/

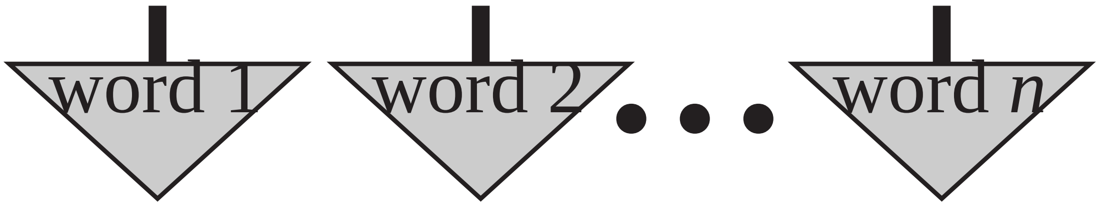
**— *the from-words-to-a-sentence process* —**

Consider meanings of **words**, e.g. as vectors (cf. Google):



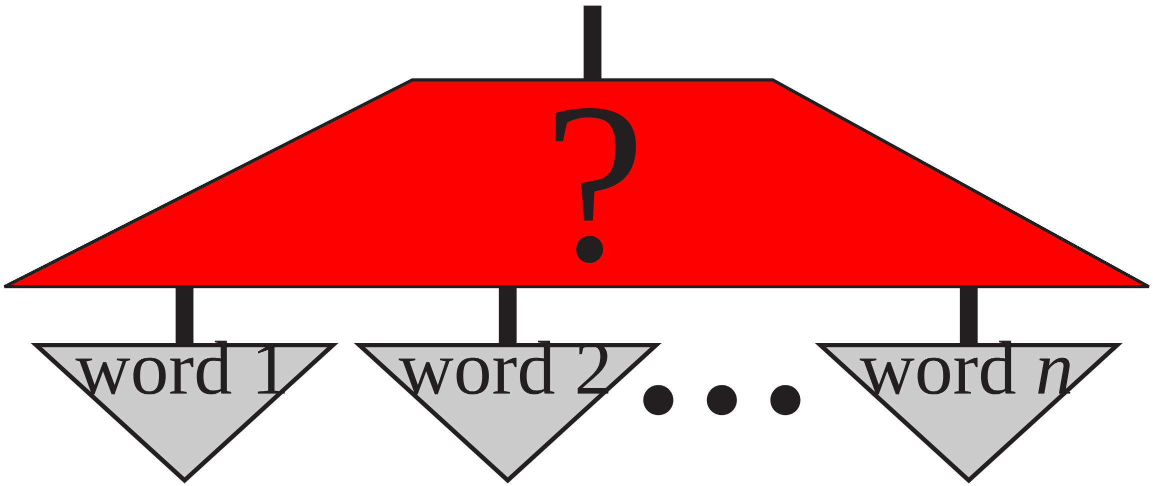
— *the from-words-to-a-sentence process* —

What is the meaning the **sentence** made up of these?



— *the from-words-to-a-sentence process* —

I.e. how do we/machines produce meanings of **sentences**?



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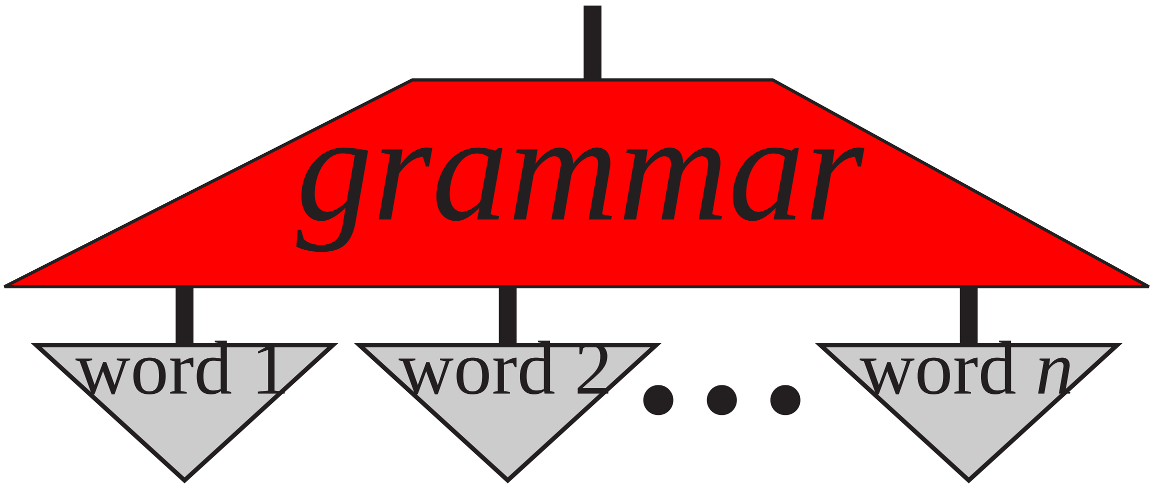
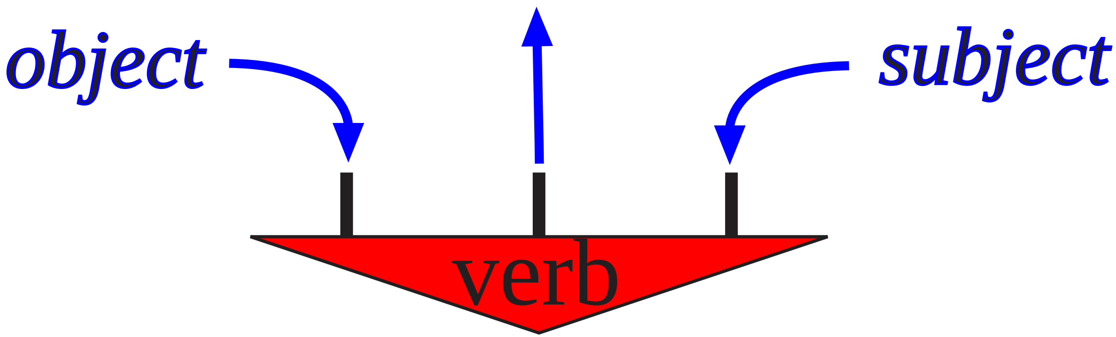


— ***the from-words-to-a-sentence process*** —

Information flow within a verb:



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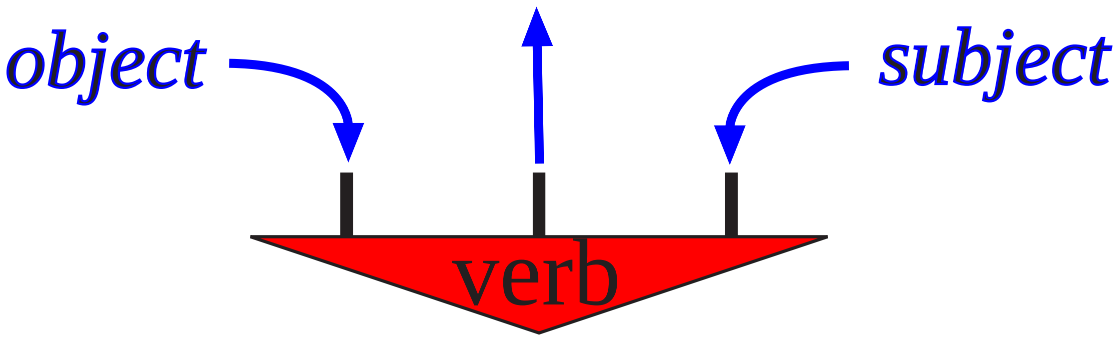
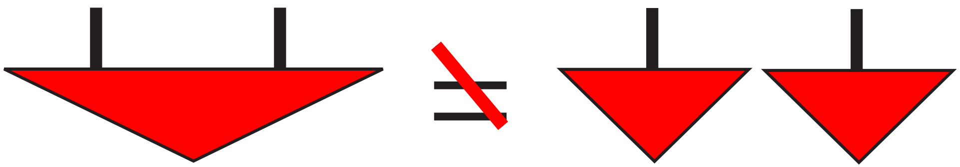
Information flow within a verb:




Again we have:



## — *pregoup grammar* —

**Lambek's residuated monoids (1950's):**

$$b \leq a \multimap c \Leftrightarrow a \cdot b \leq c \Leftrightarrow a \leq c \multimapinv b$$

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$$a \cdot (a \multimap c) \leq c \leq a \multimap (a \cdot c)$$

$$(c \multimapinv b) \cdot b \leq c \leq (c \cdot b) \multimapinv b$$

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**Lambek's pregroups (2000's):**

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**Lambek's pregroups (2000's):**

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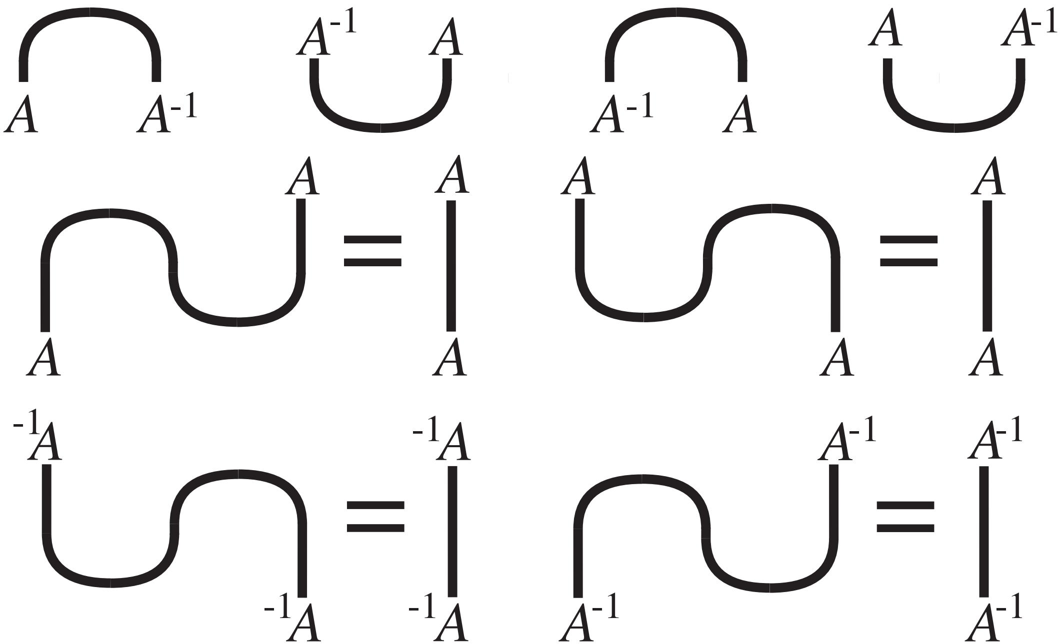

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**Diagrammatic type reduction:**

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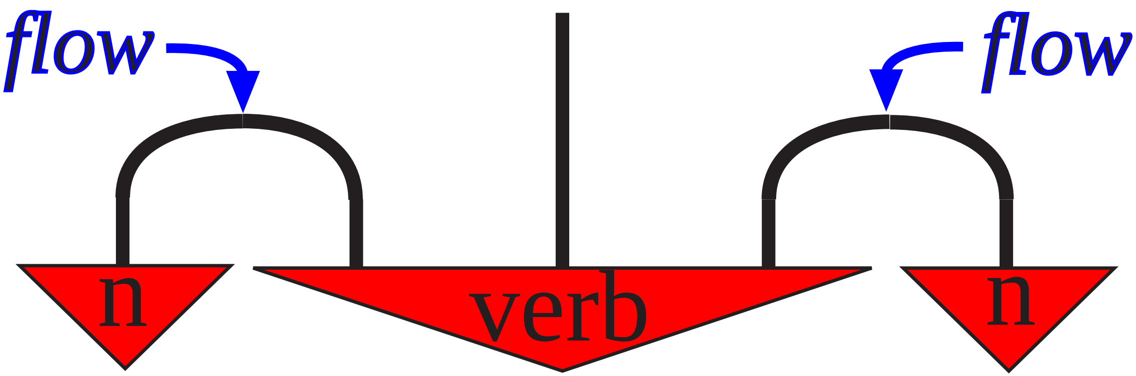
$$n \cdot {}^{-1}n \cdot s \cdot n^{-1} \cdot n \leq 1 \cdot s \cdot 1 \leq s$$

**Diagrammatic meaning:**



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1. Perform type reduction:

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3. Apply this map to tensor of word meaning vectors:

$$f\left(\overrightarrow{v}_1 \otimes \ldots \otimes \overrightarrow{v}_n\right)$$

$$— \overrightarrow{Alice} \otimes \overrightarrow{does} \otimes \overrightarrow{not} \otimes \overrightarrow{like} \otimes \overrightarrow{Bob} —$$

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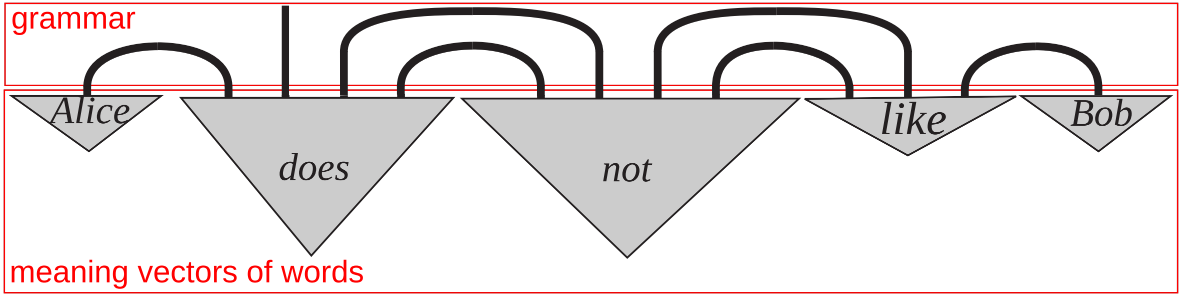
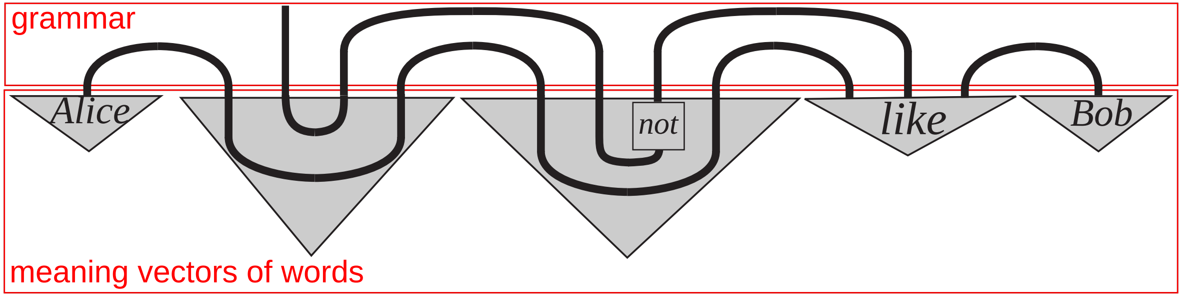


$$— \; \overrightarrow{Alice} \otimes \overrightarrow{does} \otimes \overrightarrow{not} \otimes \overrightarrow{like} \otimes \overrightarrow{Bob} \; —$$



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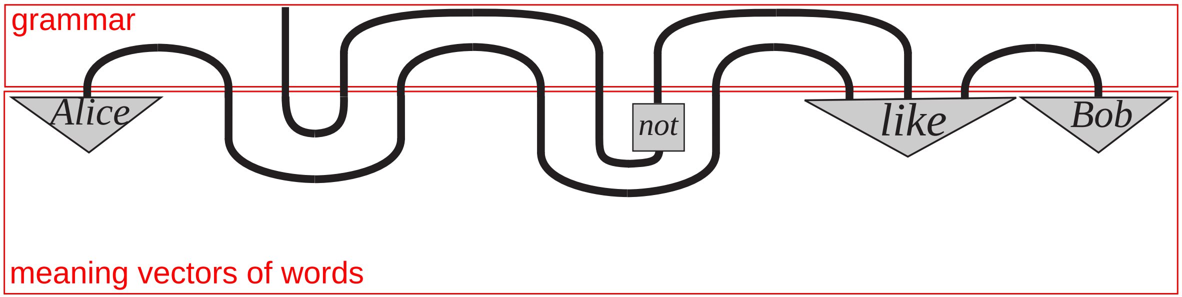

$$
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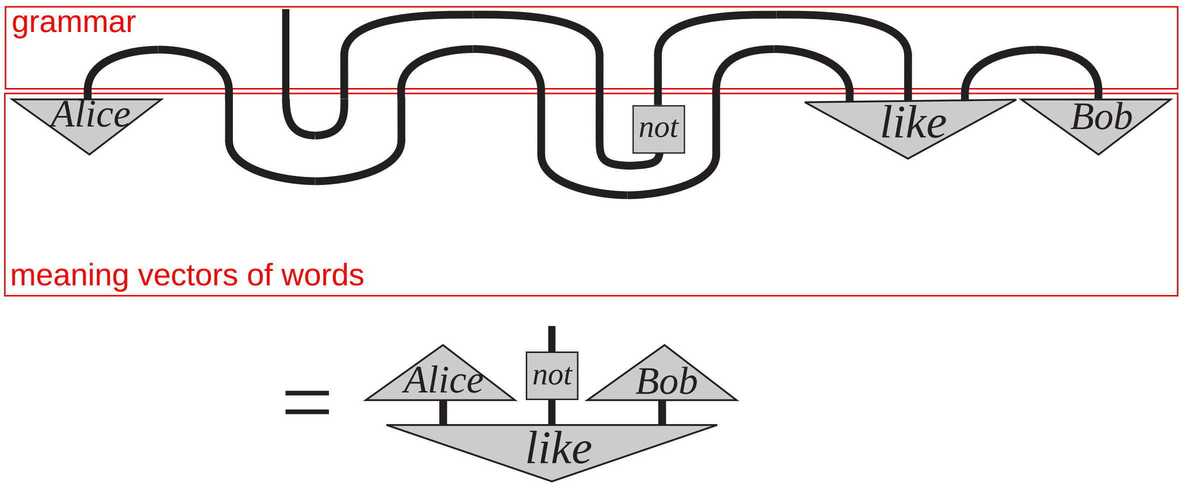
$$



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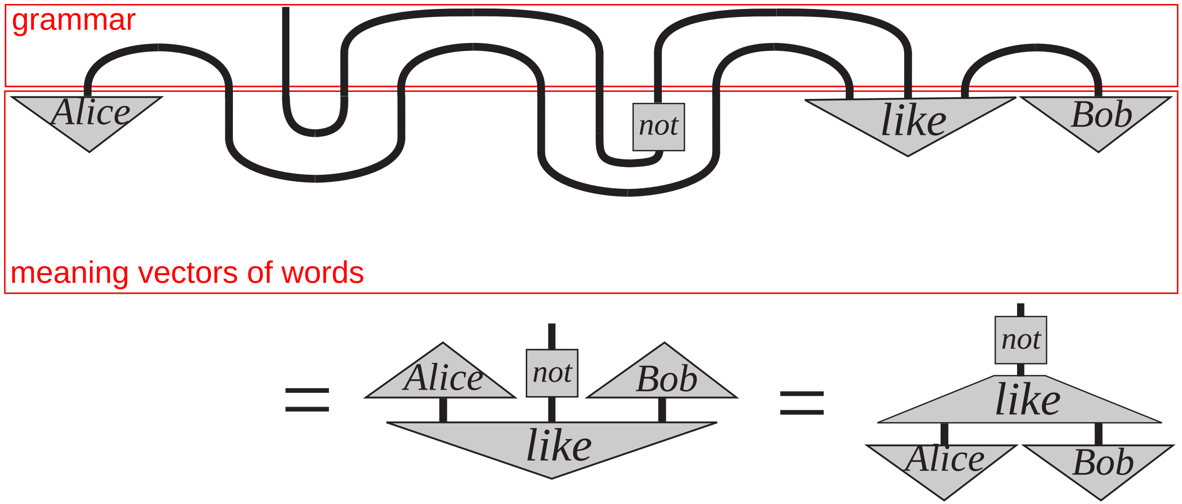


Using:

## — *experiment: word disambiguation* —

E.g. what is "saw"" in: "Alice saw Bob with a saw".

| Model | High | Low | $\rho$ |
|---|---|---|---|
| Baseline | 0.47 | 0.44 | 0.16 |
| Add | 0.90 | 0.90 | 0.05 |
| Multiply | 0.67 | 0.59 | 0.17 |
| **Categorical (1)** | **0.73** | **0.72** | **0.21** |
| **Categorical (2)** | **0.34** | **0.26** | **0.28** |
| UpperBound | 4.80 | 2.49 | 0.62 |

Edward Grefenstette & Mehrnoosh Sadrzadeh (2011) *Experimental support for a categorical compositional distributional model of meaning*. Accepted for: Empirical Methods in Natural Language Processing (EMNLP'11).

# — *Frobenius algebras* —

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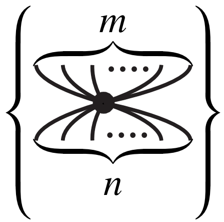
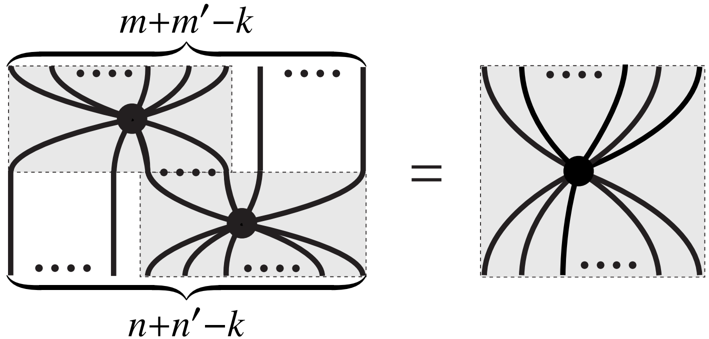
‘*spiders*’ =




such that, for $k > 0$:





BC & Dusko Pavlovic (2007) *Quantum measurement without sums*. In: Mathematics of Quantum Computing and Technology. `quant-ph/0608035`

BC, Dusko Pavlovic & Jamie Vicary (2008) *A new description of orthogonal bases*. Mathematical Structures in Computer Science. `0810.0812`

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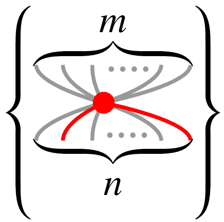
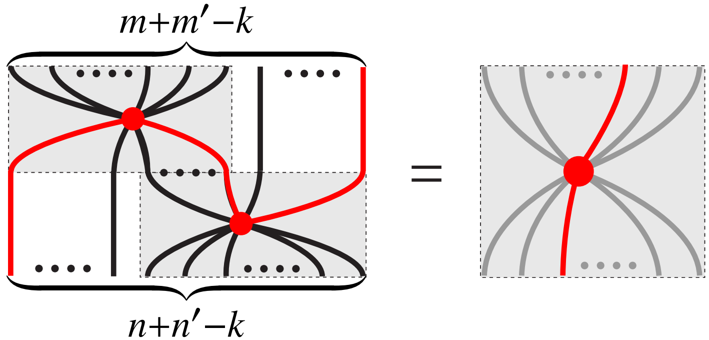
‘*spiders*’ = 


such that, for $k > 0$:

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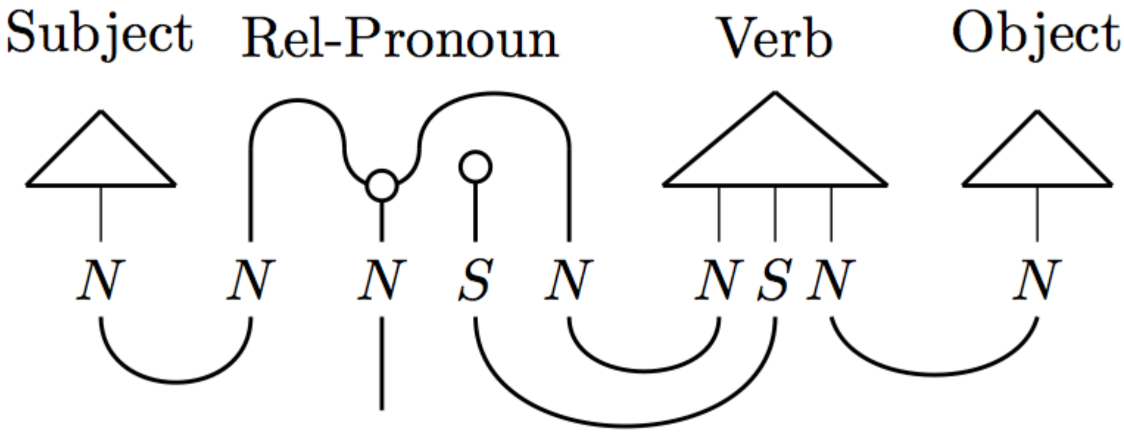
BC, Dusko Pavlovic & Jamie Vicary (2008) *A new description of orthogonal bases*. Mathematical Structures in Computer Science. `0810.0812`

## — *Frobenius algebras* —

**Language-meaning**:

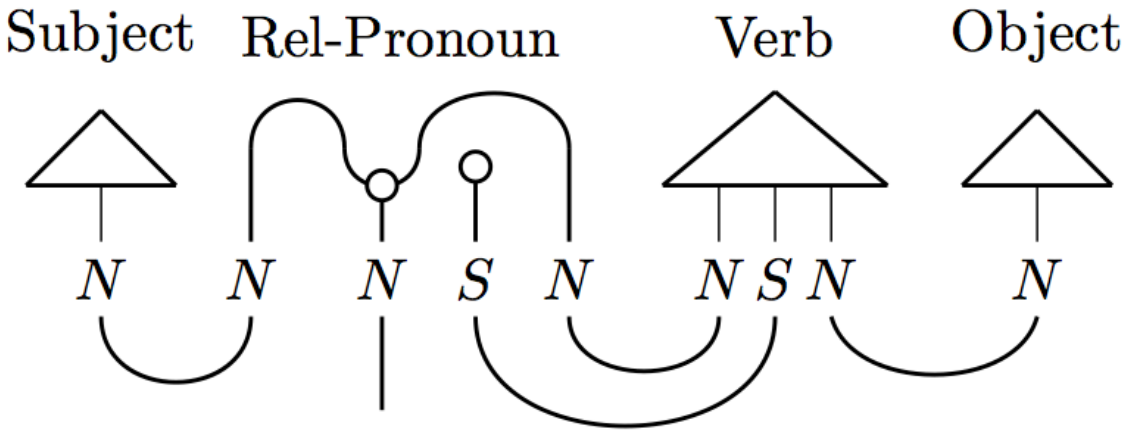



(the) man who Alice hates


Stephen Clark, BC and Mehrnoosh Sadrzadeh (2013) *The Frobenius Anatomy of Relative Pronouns*. MOL '13.

# — *Frobenius algebras* —

**Language-meaning**:

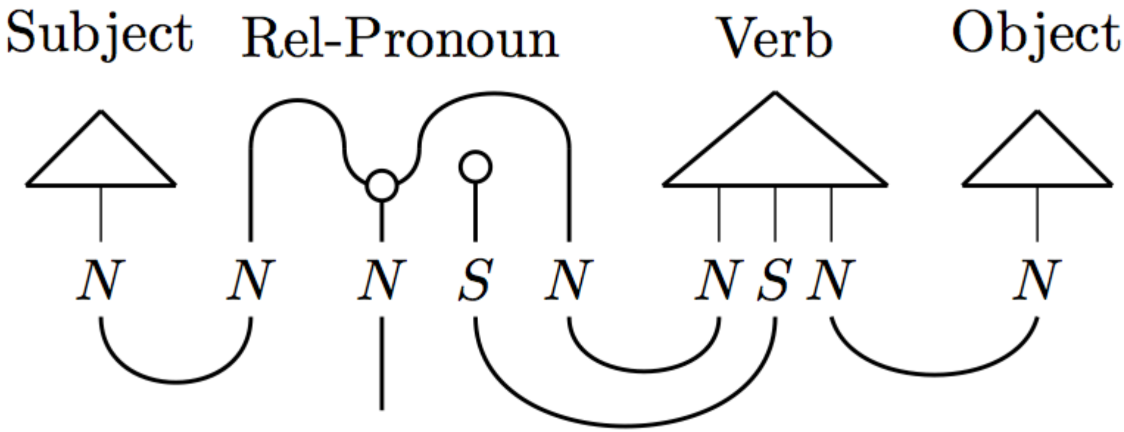



(the) man who Alice hates =


Stephen Clark, BC and Mehrnoosh Sadrzadeh (2013) *The Frobenius Anatomy of Relative Pronouns*. MOL '13.

# — *Frobenius algebras* —

**Language-meaning**:




(the) man who Alice hates = Bob

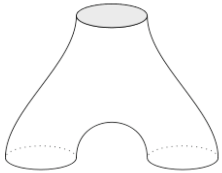
---

Stephen Clark, BC and Mehrnoosh Sadrzadeh (2013) *The Frobenius Anatomy of Relative Pronouns*. MOL '13.

# — MATHS —

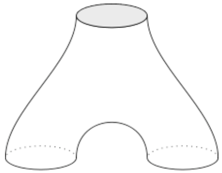
# — MATHS —

“Topological” QFT (Atiyah ’88):

$F$ ::  $\mapsto \quad f : V \otimes V \to V$

# — MATHS —

"Topological" QFT (Atiyah '88):

$F ::$  $\mapsto \quad f : V \otimes V \to V$

"Grammatical" QFT:

$F ::$  $\mapsto \quad \left(\sum_i \langle ii|\right) \otimes \mathrm{id} \otimes \left(\sum_i \langle ii|\right)$

# — MUSIC —

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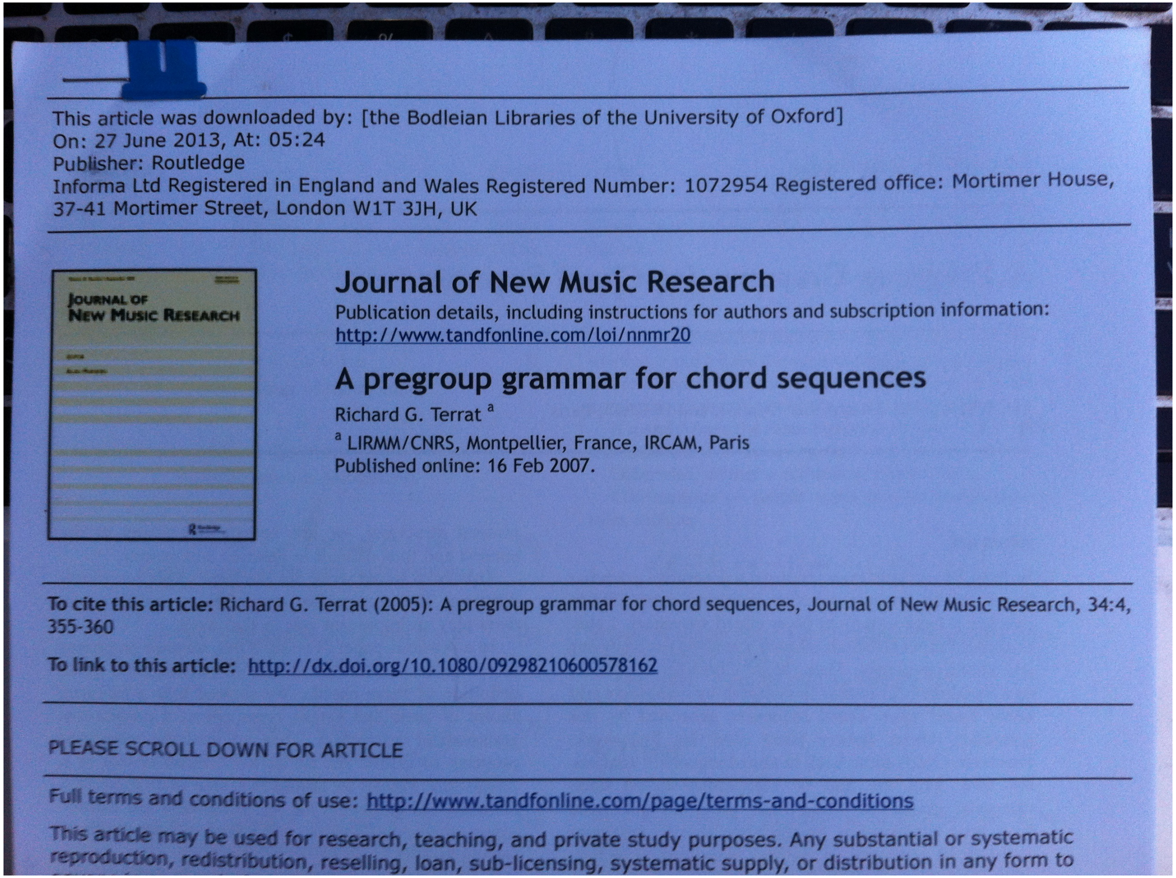



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http://www.tandfonline.com/loi/nnmr20

## A pregroup grammar for chord sequences

Richard G. Terrat [a]
[a] LIRMM/CNRS, Montpellier, France, IRCAM, Paris
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