Sliders
SDF 60th Birthday Celebration

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Remeniscences

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Lovely to do research all day!

So, what are sliders?

Pulled Pork Sliders

Chicken Sliders

Hamburger Sliders

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In mathematics, sliders are formally known as *indiscernibles*.

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Since that initial introduction, indiscernibles keep sliding into key positions in my work.

Sy and I were interested in the following:

Problem. Given models $V \subseteq W$ of ZFC, when does having a new subset of κ in $W \setminus V$ make $(\mathcal{P}_{\kappa^+}(\lambda))^W \setminus (\mathcal{P}_{\kappa^+}(\lambda))^V$ stationary in W? i.e. When is the *ground model co-stationary*?

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$S \subseteq \mathcal{P}_\kappa(\lambda)$ is *stationary* if S meets every club set.

Background

[Abraham/Shelah 1983]: ccc forcings adding a new subset of $\aleph_0$

[Gitik 1985]: models $V \subseteq W$ where W has a new subset of $\aleph_0$

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This was the beginning of our work on finding the equiconsistency of co-stationarity of the ground model and broader work in which indiscernibles play an important role.

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$\langle X, <\rangle$ is a set of *indiscernibles* for $\mathcal{M}$ iff
for all $\varphi(v_1, \dots, v_n)$ in the language of $\mathcal{M}$,
for all $x_1 < \cdots < x_n$ and $y_1 < \cdots < y_n$ in X,

$$\mathcal{M} \models \varphi[x_1, \dots, x_n] \text{ iff } \mathcal{M} \models \varphi[y_1, \dots, y_n].$$

α-Erdős cardinals

κ is *α-Erdős* if for each structure $\mathcal{M}$ in a countable language with universe κ (endowed with Skolem functions), for each club $C \subseteq \kappa$ there is a set $I \subseteq C$ of order type α such that

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Thm. [Dobrinen/Friedman 06] Suppose that in V, $\lambda > \kappa$, κ is regular, and λ is κ-Erdős. Let $\mathbb{C}_\kappa$ be κ-Cohen forcing
(or any $(\lambda, \lambda, \kappa)$-distributive partial ordering adding a new subset of κ).

Then $(\mathcal{P}_{\kappa^+}(\mu))^{V^{\mathbb{C}_\kappa}} \setminus (\mathcal{P}_{\kappa^+}(\mu))^V$ is stationary in $V^{\mathbb{C}_\kappa}$ for all $\mu \geq \lambda$.

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Pushing the κ down to smaller cardinals involved gleaning tree coding from some work of [Baumgartner 1991].

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Thm (Global Gitik). [Dobrinen/Friedman 06] The following are equiconsistent:

1. There is a proper class of ω_1-Erdős cardinals.
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Indiscernibles were also important in our work on the internal consistency strength of co-stationarity of the ground model [Dobrinen/Friedman 2008].

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The proofs of such theorems heavily involve the Silver indiscernibles for building the generics.

And now for something discernibly different

Ramsey Theory

Ramsey Theorem. For each $k, n \geq 1$ and coloring $c : [\omega]^k \to n$, there is an infinite $M \subseteq \omega$ such that c restricted to $[M]^k$ monochromatic. That is, M is homogeneous.

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What about colorings into infinitely many colors?

Order Indiscernibility

Erdős-Rado Canonization Theorem. For each $k \geq 1$ and each equivalence relation E on $[\omega]^k$, there is an infinite $M \subseteq \omega$ such that $\mathrm{E} \restriction [M]^k$ is *canonical*; i.e. $\mathrm{E} \restriction [M]^k$ is given by E^k_I for some $I \subseteq k$.

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The proofs of these theorems involve sliding of points between fixed points; in essence, indiscernibility.

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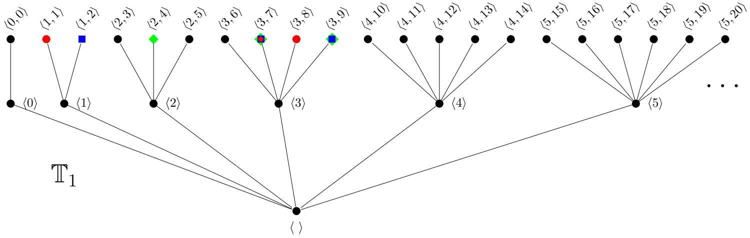
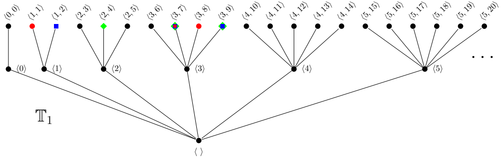
The Next Topological Ramsey Space: $\mathcal{R}_1$ [D/T 1]



$X \in \mathcal{R}_1$ iff X is a subtree of $\mathbb{T}_1$ and $X \cong \mathbb{T}_1$.
For $X, Y \in \mathcal{R}_1$, $Y \leq_1 X$ iff $Y \subseteq X$.



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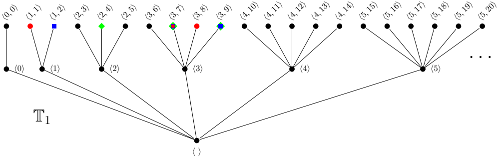

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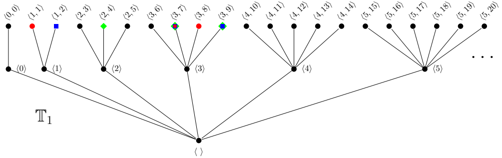
Numbers of Canonical Equivalence Relations on Finite Rank Barriers [D/T 1]



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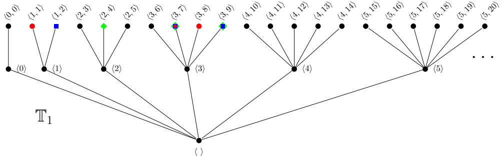

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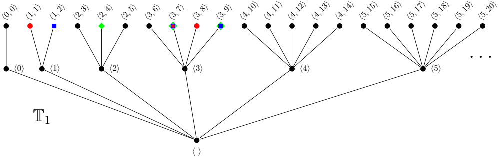

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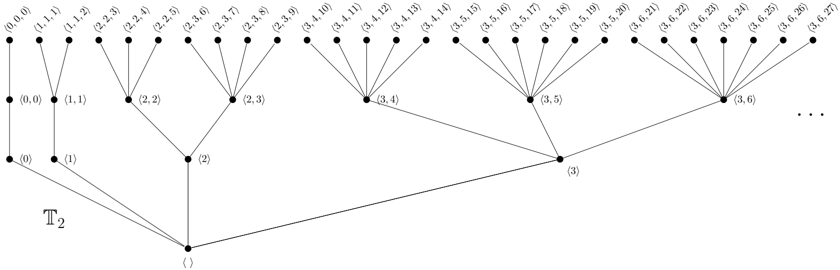
k-approximations: $(2^1 + 1)(2^2 + 1) \cdots (2^k + 1)$.

The space $\mathcal{R}_2$



Theorems in [Dobrinen/Todorcevic 2]

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Ramsey-classification theorems for equivalence relations on barriers were used to classify all Rudin-Keisler isomorphism types of ultrafilters within the Tukey type of ultrafilters with weak partition properties.

Classification of Tukey vs Rudin-Keisler for ultrafilters

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Thm. [Dobrinen/Todorcevic 1,2] For each $\alpha < \omega_1$, there is an ultrafilter $\mathcal{U}_\alpha$ which is a rapid p-point, has partition properties, and the cofinal types below it form a chain of order-type $(\alpha + 1)^*$.

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Moreover, the isomorphism types within these cofinal types are completely classified as tree ultrafilters, where branching occurs according to p-points from a precise countable collection determined by the canonization theorem.

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More on this in Barcelona.

References

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Happy 60th Birthday Sy!

Happy wishes as you slide into the next decade!