KURT GÖDEL RESEARCH CENTER FOR MATHEMATICAL LOGIC

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INVITATION

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(SUB)SYSTEMS OF SECOND-ORDER SET THEORY

Abstract:

Much of set-theoretic practice concerns questions that are, at first blush, second-order in content. The study of the construction of of certain kinds of *maximality principle* (such as Friedman's Inner Model Hypothesis), are all naturally understood as concerning second-order classes rather than sets.

Understandably, given the pleasant metalogical properties of first-order ZFC, many set theorists work hard to render their second-order interests in first-order terms. However, increasingly set theorists have become engaged in questions that are *greater than first-order* (good examples being the results concerning embeddings from inner models to the universe, the study of open determinacy for class games, and the consistency of the IMH).

In the philosophical literature, there is a debate concerning how to characterise proper classes within the framework of there being a unique, maximal proper class model of set theory. Traditionally, talk of proper classes in set theory was understood as shorthand for statements definable in terms of first-order formulae with parameters. However, in the last forty years, philosophical conceptions of proper classes have been proposed which aim to capture this essentially second-order character of set-theoretic practice. In particular, Boolos and Uzquiano develop a paraphrase in terms of plural quantification, where Horsten and Welch provide a mereological conception of proper classes.

In this paper, we examine what can be extracted from particular philosophical conceptions of classes, focussing on the plural conception. First (§1), we provide some motivating considerations for the choice of the plural paraphrase. In particular, we argue that the plural paraphrase meshes better with the *foundational* role many have seen for set theory. Next (§2), we note that this conception of classes has been viewed to motivate one of two class theories, either (1.) MK or (2.) NBG. We argue that this is a false dichotomy; just as in the case of subsystems of second-order arithmetic, we should expect there to be various philosophical motivations for different strengths of class theories both intermediate between NBG and MK, and above MK. Finally (§3), we examine some of the relevant technical literature, and draw some philosophical conclusions. We argue that naturalistic considerations motivate the use of *some* non-definable class talk. In particular, we argue for two conclusions (1.) Π^1_1-comprehension for classes is motivated by its having many independently justified consequences made clear in the work of Gitman and Hamkins, and (2.) given a stronger naturalism we can justify the use of strong choice principles for classes extending MK on the basis of work of Gitman. We conclude that a detailed philosophical and mathematical study of (sub)systems of second-order set theory is in order, including some which extend MK.

THURSDAY, JUNE 16, 2016
Tea at 3:30pm in the KGRC meeting room (room 104)
Talk at 4:00pm in the KGRC lecture room (room 101)
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