INVITATION

**PETER NYIKOS**
**(University of South Carolina, Columbia, USA)**

# CARDINALITY RESTRICTIONS ON SOME KINDS OF LOCALLY COMPACT SPACES

Abstract:

In what follows, "space" means "Hausdorff ($T_2$) topological space."

Some of the theorems and problems to be discussed include:

Theorem 1. It is ZFC-independent whether every locally compact, $\omega_1$-compact space of cardinality $\aleph_1$ is the union of countably many countably compact spaces.

['$\omega_1$-compact' means that every closed discrete subspace is countable. This is obviously implied by being the union of countably many countably compact spaces, but the converse is not true.]

Problem 1. Is it consistent that every locally compact, $\omega_1$-compact space of cardinality $\aleph_2$ is the union of countably many countably compact spaces?

Problem 2. Is ZFC enough to imply that there is a normal, locally countable, countably compact space of cardinality greater than $\aleph_1$?

Problem 3. Is it consistent that there exists a normal, locally countable, countably compact space of cardinality greater than $\aleph_2$?

The spaces involved in Problem 2 and Problem 3 are automatically locally compact, because regularity is already enough to give every point a countable countably compact (hence compact) neighborhood.

Problem 4 [Problem 5]. Is there an upper bound on the cardinalities of regular [resp. normal], locally countable, countably compact spaces?

Theorem 2. The axiom $\square_{\aleph_1}$ implies that there is a normal, locally countable, countably compact space of cardinality $\aleph_2$.

The statement in Theorem 1 was shown consistent by Lyubomyr Zdomskyy, assuming $\mathfrak{p} > \aleph_1$ plus P-Ideal Dichotomy (PID). Counterexamples have long been known to exist under $\mathfrak{b} = \aleph_1$, under $\clubsuit$, and under the existence of a Souslin tree.

Theorem 2 may be the first application of $\square_{\aleph_1}$ to construct a topological space whose existence in ZFC is unknown.