KURT GÖDEL RESEARCH CENTER FOR MATHEMATICAL LOGIC

UNIVERSITÄT WIEN

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INVITATION

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THE EXACT STRENGTH OF THE CLASS FORCING THEOREM

Abstract:

Gödel–Bernays set theory GBC proves that sufficiently nice (i.e. pretame) class forcings satisfy the forcing theorem—that is, these forcing notions $\mathbb{P}$ admit forcing relations $\Vdash_{\mathbb{P}}$ satisfying the recursive definition of the forcing relation. It follows that statements true in the corresponding forcing extensions are forced and forced statements are true. But there are class forcings for which having their forcing relation exceeds GBC in consistency strength. So GBC does not prove the forcing theorem for all class forcings. This is in contrast to the well-known case of set forcing, where ZFC proves the forcing theorem for all set forcings. On the other hand, stronger second-order set theories such as Kelley–Morse set theory KM prove the forcing theorem for all class forcings, providing an upper bound. What is the exact strength of the class forcing theorem?

I will show that, over GBC, the forcing theorem for all class forcings is equivalent to $\mathsf{ETR}_{\mathrm{Ord}}$ the principle of elementary transfinite recursion for recursions of height Ord. This is equivalent to the existence of Ord-iterated truth predicates for first-order truth relative to any class parameter; which is in turn equivalent to the existence of truth predicates for the infinitary languages $\mathcal{L}_{\mathrm{Ord},\omega}(\in, A)$ allowing any class parameter A. This situates the class forcing theorem precisely in the hierarchy of theories between GBC and KM.

This is joint work with Victoria Gitman, Joel Hamkins, Peter Holy, and Philipp Schlicht.

TUESDAY, SEPTEMBER 26, 2017
Tea at 3:30pm in the KGRC meeting room (room 104)
Talk at 4:00pm in the KGRC lecture room (room 101)
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