AXIOMATIC SET THEORY 1. SUMMER SEMESTER 2015.

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The main aim of the course will be to reach some basic facts about forcing, e.g., the consistency of the negation of the CH.

Content of the course (= material required for the exam) is a *proper* part of Chapters I-II and IV of the Kunen's book [1]. I.e., if you learn all these chapters you will know more than needed to pass the exam with the best note. During most of the lectures I will follow this book. My plan is to update this file every week. In particular, it will contain an up-to-date list of the topics which have already been covered. Should I decide to deviate from the Kunen's book, I will write some lecture notes.

Another book containing most of the information which I plan to present is [3].

The Exam will be oral.

You can pass the exam on either of the following days:

2. 6.07.2015, 10:00-12:00.

3. 21.09.2015, 10:00-12:00

Please send me a short e-mail at least 2 days in advance!

Should you prefer to have an exam on some other day, any time which doesn't contradict the rules of the University is suitable for me. Again, an e-mail a couple of days in advance is needed!

Schedule.

Monday, 10.00 – 12.20;
First lecture: 09.03.2015. Last lecture: 29.06.2015

Thanks to the participants of my previous courses I have some parts of [1] scanned. Send me an e-mail if you would like to have them.

All necessary facts from mathematical logic we will use are available in
http://home.mathematik.uni-freiburg.de/ziegler/skripte/logik.pdf

Language: English.

What have we already learned

- Lecture 1, 09.03.2015.
  Chapter 1 until Definition I.6.21. We proceeded rather quickly and without many details because this part was presented in the "Introduction to Mathematical Logic" course, see
  `http://www.logic.univie.ac.at/~lzdomsky/iml_ws2014.pdf`
- Lecture 2, 16.03.2015.
  Chapter I until Lemma 1.8.5.
- Lecture 3, 23.03.2015.
  Chapter I until Lemma 1.9.23.

- Lecture 4, 13.04.2015.
  Chapter I until Theorem 1.10.4.
- Lecture 5, 20.04.2015.
  Chapter I until Theorem 1.12.1.
- Lecture 6, 27.04.2015.
  Chapter I until Theorem 1.13.12.
- Lecture 7, 04.05.2015.
  Chapter I until Definition 1.13.30.
- Lecture 8, 11.05.2015.
  Chapter I until Lemma 1.15.11.
- Lecture 9, 18.05.2015.
  We finished with Chapter I. We covered only those pieces of I.14-I.17 which are to be used later. You are of course welcome to read these paragraphs completely.
- Lecture 10, 01.06.2015.
  Chapter II until Lemma II.4.3.
- Lecture 11, 08.06.2015.
  Chapter II until Corollary II.5.4. I've often referred you to my "Introduction to Math. Logic" course and to "Tarski's definability of truth" while using arguments requiring something like "for all formulas ϕ". There is a very efficient and simple way how to do all such proofs in this course avoiding essentially formal logic. It is described in the first paragraph of
  `http://www.logic.univie.ac.at/~lzdomsky/F.pdf`.
  Please note that I am strongly encouraging you to understand the logic behind all these arguments, and offer the above detour just for curiocity.
- Lecture 12, 15.06.2015.
  Chapter II until Lemma II.6.22.
- Lecture 13, 22.06.2015 (plus one extra lecture on 26.06.2015).
  We have proved Theorem II.6.26 and switched to [2, Chapter VII,§2][1]. We reached Theorem 3.5 on p. 197.
- Lecture 14, 29.06.2015.
  We proved the consistency of $2^\omega > \omega_1$ assuming the consistency of ZFC, following the exposition in [2].
  In the lecture we assumed that M, the ground model, is a ctm of ZFC. This was more than needed, see [2, Chapter VII, §1 and §9(1a)].

References

[1] Kunen, K., *Set theory.* Studies in Logic (London), 34. College Publications, London, 2011.

[2] Kunen, K., *Set theory. An introduction to independence proofs.* Studies in Logic and the Foundations of Mathematics, 102. North-Holland Publishing Co., Amsterdam-New York, 1980. xvi+313 pp.

[1] Previouly we always followed [1].

[3] Jech, T., *Set theory.* The third millennium edition, revised and expanded. Springer Monographs in Mathematics. Springer-Verlag, Berlin, 2003. xiv+769 pp.

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