READING COURSE IN SET THEORY. WINTER SEMESTER 2015–2016.

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The main aim of the course will be to reach some interesting applications of forcing, e.g., in the set theory of reals.

Content of the course (= material required for the exam).
This course will continue approximately at the point where the course
Axiomatische Mengenlehre 1, I taught in the previous semester
(`http://www.logic.univie.ac.at/ lzdomsky/set_theory_1_ss2015.pdf`),
left off. We will start with iterated forcing, MA, then move to proper forcing, along with applications. Among others, I plan to cover the Laver's result that consistently all strong measure zero sets are countable.

The Exam will be oral.
You can pass the exam on either of the following days:
1. 29.01.2016, during the last lecture.
2. 12.02.2016, 10:00-12:00.
3. To be announced later.
Please send me a short e-mail at least 2 days in advance!
Should you prefer to have an exam on some other day, any time which doesn't contradict the rules of the University is suitable for me. Again, an e-mail a couple of days in advance is needed!

Schedule. Friday, 15:00–17:20.

Thanks to the participants of my previous courses I have some parts of [1, 4, 5, 3] scanned. Send me an e-mail if you need them.

All necessary facts from mathematical logic we will use are available in
http://home.mathematik.uni-freiburg.de/ziegler/skripte/logik.pdf

Language: English.

What have we already learned and reading material for the next lecture

- *Lecture 1, 02.10.2015*
  Done: We have repeated some basic facts about forcing covered in my "Set Theory 1" course mentioned above.
  To be read: [5, pp. 211 - 223], from the beginning of §6 until Boolean-valued models.
- *Lecture 2, 09.10.2015*
  Done: All suggested to be read except for 7.12, 7.13 on [5, pp. 222]. We'll come back to these later.
  To be read: [5, Ch. VIII, §5] until Lemma 5.14.

- *Lecture 3, 16.10.2015*
  Done: We reached Definition 5.9 on [5, p. 273] and discussed it.
  To be read: [5, Ch. VIII, §6].
- *Lecture 4, 23.10.2015, by Stefan Hoffelner.*
  Done: We proved the consistency of the MA.
  To be read: [4, pp. 379–385] and an initial part of [1] until the proof
  that the countable support iteration of proper posets is proper.
- *Lecture 5, 11.12.2015.*
  Done: Following mainly [1], we proved some basic properties of
  proper forcing.
  To be read: [1, Definition 2.3 and §2.1].
- *Lecture 6, 18.12.2015.*
  Done: Again following [1], we proved that countable support itera-
  tions of proper posets is again proper, and such iterations of length
  $\leq \omega_2$ have ω_2-c.c. provided that CH holds in the ground model and
  the iterands have size $\leq \omega_1$.
  To be read: [6] until p. 159.
- *Lecture 7, 08.01.2016.*
  Done: Following [6], we have established some basic properties of
  the Laver forcing.
  To be read: [6] till the end.
- *Lecture 8, 15.01.2016.*
  Done: We reached [6, Lemma 6].
  To be read: [2, pp.23-27]. This seems to be the best esxplanation
  of how to treat an iteraion $\mathbb{P}_\alpha$ of length α as a two step iteration
  $\mathbb{P}_\beta * \mathbb{P}_{\beta,\alpha}$.
- *Lecture 9, 22.01.2016.*
  Done: We've finished with the proof of [6, Lemma 12].
  To be read: The rest of [6].
- *Lecture 10, 29.01.2016.*
  Done: [6] till the end.

References

[1] Abraham, Uri, *Proper forcing.* Handbook of set theory. Vols. 1, 2, 3, 333–394, Springer, Dordrecht, 2010.
[2] Baumgartner, J.E., *Iterated forcing,* in: *Surveys in set theory* (A.R.D. Mathias ed.), London Math. Soc. Lecture Note Ser., 87, Cambridge Univ. Press, Cambridge, 1983, p. 1–59.
[3] Jech, T., *Set theory.* The third millennium edition, revised and expanded. Springer Monographs in Mathematics. Springer-Verlag, Berlin, 2003. xiv+769 pp.
[4] Kunen, K., *Set theory.* Studies in Logic (London), 34. College Publications, London, 2011.
[5] Kunen, K., *Set theory. An introduction to independence proofs.* Studies in Logic and the Foundations of Mathematics, 102. North-Holland Publishing Co., Amsterdam-New York, 1980. xvi+313 pp.
[6] Laver, R., *On the consistency of Borel's conjecture,* Acta Math. **137** (1976), 151–169.

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