Sealed Trees and the Perfect Subtree Property for Weakly Compact Cardinals

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κ-Trees and Upper Bounds

A First Lower Bound and Sealed Trees

Below Woodin cardinals

A Non-domestic Mouse

κ-Trees

Definition

Let κ be a regular cardinal. A tree T of height κ is called a *normal κ-tree* if

- each level of T has size $<\kappa$,
- every node splits,
- for every $t \in T$ and $\alpha < \kappa$ above the height of t, there is some t' of level α in T such that $t <_T t'$, and
- for every limit ordinal $\alpha < \kappa$ and every branch up to α there is at most one least upper bound in T.

The Branch Spectrum

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$$\mathfrak{S}_\kappa = \{|[T]| \mid T \text{ is a normal } \kappa\text{-tree}\}.$$

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- $\mathfrak{S}_\omega = \{2^{\aleph_0}\}$.
- For $\kappa > \omega$, there are no κ-Kurepa trees iff $\max(\mathfrak{S}_\kappa) = \kappa$.
- For $\kappa > \aleph_1$ of uncountable cofinalty, tree property holds at κ iff $\min(\mathfrak{S}_\kappa) = \kappa$.

Upper Bounds

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The following gives an upper bound.

Proposition

Let κ be $<\mu$-supercompact, where μ is strongly inaccessible. Then, there is a forcing extension in which κ is weakly compact, $\mathfrak{S}_\kappa = \{\kappa, \kappa^{++}\}$.

Proof idea: Consider $\mathrm{Col}(\kappa, <\mu) \times \mathrm{Add}(\kappa, \mu^+)$.

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Question

Is this optimal?

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Strongly sealed trees with κ many branches exist in ZFC: Take $T \subseteq 2^{<\kappa}$ to be the tree of all x such that $\{\alpha \in \mathrm{dom}(x) \mid x(\alpha) = 1\}$ is finite.

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Question

How about strongly sealed κ-trees with at least κ^+ many branches?

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A Sealed Tree in K

Theorem (Hayut, M.)

Let us assume that there is no inner model with a Woodin cardinal. Then for every inaccessible cardinal κ, there is a strongly sealed κ-tree with exactly $(\kappa^+)^K$ many branches. In particular, if κ is weakly compact, then there is a strongly sealed tree on κ with κ^+ many branches.

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- **Nodes:** $\langle \bar{M}, \bar{x} \rangle$, where $\bar{M} = \mathrm{trcl}(\mathrm{Hull}^{K_{\kappa^+}}(\rho \cup \{x\}))$ for some $\rho < \kappa$, $x \in K_{\kappa^+} \cap {}^{\kappa}2$ and x collapses to $\bar{x}$.

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- **Tree order:** $\langle M_0, x_0 \rangle \leq \langle M_1, x_1 \rangle$ if there is some ordinal ρ such that $M_0 = \mathrm{trcl}(\mathrm{Hull}^{M_1}(\rho \cup \{x_1\}))$ and x_1 collapses to x_0.

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Claim

$\mathbb{T}(K_{\kappa^+})$ is a tree of height κ with at least $(\kappa^+)^K$ many branches.

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- Argue that each branch in V is in fact already in K, so $|[T]| = (\kappa^+)^K$ (use maximality and universality of K).

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- A more careful analysis yields $b \in K$.

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Theorem (Hayut, M.)

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- Use forcing absoluteness to see that T is strongly sealed.
- Use covering to obtain $(\kappa^+)^K = (\kappa^+)^V$.

Sealed Trees in the context of Woodin cardinals

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Observation

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Why? Woodin's stationary tower forcing with critical point κ^+ will introduce new branches to any κ-tree T, while preserving the regularity of κ, as well as many large cardinal properties of κ.

The κ-Perfect Subtree Property

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Lemma (Folklore)

Let κ be a cardinal. The following are equivalent for a tree T of height κ:

1. *T has a perfect subtree.*
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Let κ be $< \mu$-supercompact, where μ is strongly inaccessible. Then, there is a forcing extension in which κ is weakly compact, $\mathfrak{S}_\kappa = \{\kappa, \kappa^{++}\}$ and the Perfect Subtree Property holds at κ.

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A Non-domestic Mouse from the κ-PSP

Theorem (Hayut, M.)

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Then there is a non-domestic mouse. In particular, there is a model of $\mathrm{ZF} + \mathrm{DC} + \mathrm{AD}_{\mathbb{R}}$.

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Proof idea:

- Consider the tree $\mathbb{T}(\mathcal{S})$ for $\mathcal{S} = \mathcal{S}(\kappa)$ the stack of mice on $K^c||\kappa$ (cf. Andretta-Neeman-Steel and Jensen-Schimmerling-Schindler-Steel).

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- $\mathbb{T}(\mathcal{S})$ has exactly $(\kappa^+)^V$ many branches (using covering as in JSSS).
- $\mathbb{T}(\mathcal{S})$ does not have a perfect subtree (argue that set of branches does not change in an $\mathrm{Add}(\kappa,1)$-generic extension of V).

How sealed is the tree?

Definition (Neeman, Steel)

Let κ be a regular cardinal. A normal tree T of height κ is *sealed* if the set of branches of T cannot be modified by set forcing $\mathbb{P}$ satisfying the following properties:

1. $\mathbb{P} \times \mathbb{P}$ does not collapse κ,
2. $\mathbb{P} \times \mathbb{P}$ preserves $\mathrm{cf}(\kappa) > \omega$, and
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Theorem

Let κ be a weakly compact cardinal and assume that there is no non-domestic premouse. Then there is a sealed κ-tree with exactly κ^+ many branches.

Looking beyond

Definition (Neeman, Steel)

A cardinal κ is Π^1_1-Woodin, if for every $A \subseteq V_\kappa$, and for every Π^1_1-statement Φ for which $\langle V_\kappa, \in, A\rangle \models \Phi$, there is a $<\kappa$-A-strong cardinal μ such that $\langle V_\mu, \in, A \cap V_\mu\rangle \models \Phi$.

Note: The least Π^1_1-Woodin cardinal is below the least Shelah cardinal.

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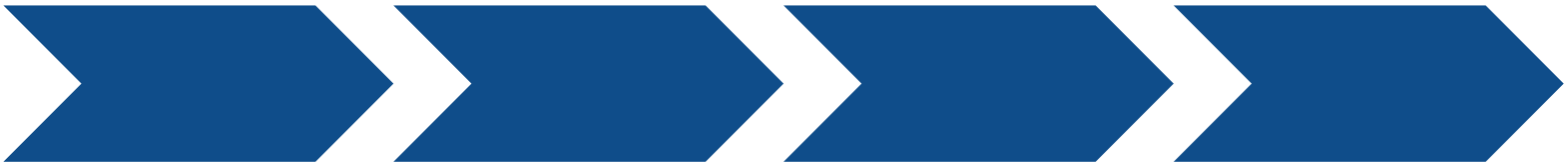
Theorem

Let κ be Π^1_1-Woodin and assume SBH_κ. Then either κ is Π^1_1-κ^+-subcompact in an inner model or there is a sealed κ-tree on κ with κ^+ many branches.



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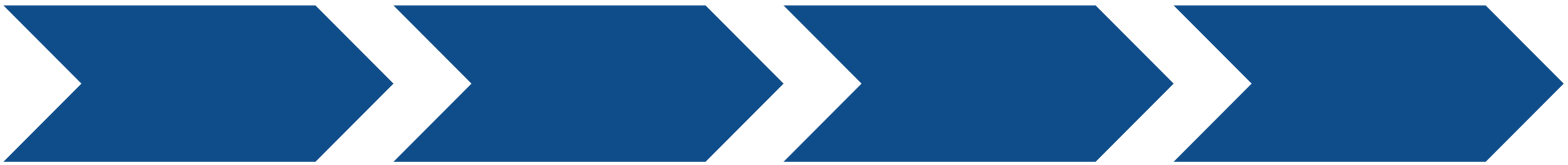

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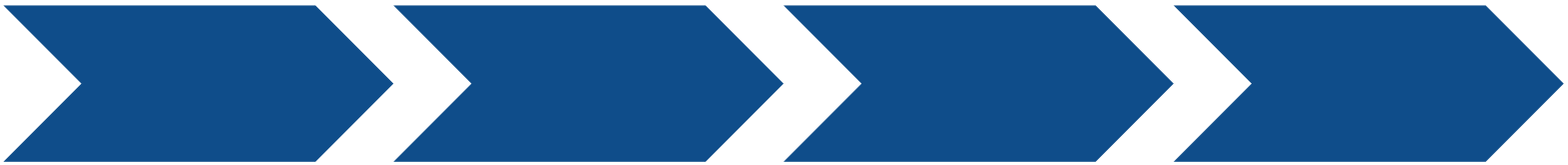
Conjecture

Let κ be a weakly compact cardinal with the Perfect Subtree Property. Then there is an inner model with a pair of cardinals $\lambda < \mu$ such that λ is $<\mu$-supercompact and μ is inaccessible.

https://arxiv.org/abs/1910.05159



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"Never say there is nothing beautiful in the world anymore. There is always something to make you wonder in the shape of a tree, the trembling of a leaf."

Albert Schweitzer

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