

# LECTURE ADVANCED TOPICS IN MATHEMATICAL LOGIC

SUMMERSEMESTER 2020

DR. SANDRA MÜLLER

The first and second part of this class cover different material.

The first part (held by Sandra Müller) covers the basic theory of determinacy. We will introduce the concept, discuss important consequences e.g. related to descriptive set theory and, if time allows, go into the connection with large cardinal theory.

Time and place: Fridays 8:45am - 11:00am in the KGRC lecture room. Please check u:find to see in which weeks there are no lectures. Starting Mar 13th, this class will be held in “home learning”. See moodle for more information.

The following is a brief summary of the content of every lecture. This is just an overview and there is no guarantee for correctness or completeness.

**Lecture 1 (Fri Mar 6th)** Introduction, infinite 2 player games, the Axiom of Determinacy AD, open and closed sets are determined (ZFC), there is a non-determined set of reals (ZFC), regularity properties, the Banach-Mazur game, statement of the characterization of meagerness via the Banach-Mazur game, proof of the corollary that under AD every set of reals has the Baire property

**Lecture 2 (Fri Mar 13th)** Proof of the characterization of meagerness via the Banach-Mazur game, the perfect set property and the Davis game, Lebesgue measurability and the covering game

**Lecture 3 (Fri Mar 20th)** Borel hierarchy, games with rules, coverings and unravelings of games, Martin’s theorem on Borel determinacy (proof except for Lemma 4.9)

**Lecture 4 (Fri Apr 3rd)** Proof of Lemma 4.9, the projective hierarchy, measurable cardinals

**Lecture 5 (Fri Apr 24th)** Martin’s theorem: if there is a measurable cardinal, then analytic games are determined, discussion of Harrington’s result: sharps from analytic determinacy

## REFERENCES

- [Kan08] KANAMORI, A.: *The Higher Infinite: Large Cardinals in Set Theory from Their Beginnings*. Springer, 2008 (Springer Monographs in Mathematics). – ISBN 9783540888666
- [Kec95] KECHRIS, A.S.: *Classical Descriptive Set Theory*. Springer, New York, 1995. – ISBN 978-1-4612-8692-9
- [KW10] KOELLNER, P. ; WOODIN, W. H.: Large Cardinals from Determinacy. In: FOREMAN, M. (Hrsg.) ; KANAMORI, A. (Hrsg.): *Handbook of Set Theory*. Springer, 2010
- [Sch14] SCHINDLER, R.: *Set Theory: Exploring Independence and Truth*. Springer International Publishing, 2014 (Universitext). – ISBN 9783319067254