A.W.F. Edwards. ***Cogwheels of the Mind: the story of Venn diagrams.*** The Johns Hopkins University Press, Baltimore and London, 2004, xvi + 109 pp.

A Venn diagram is a familiar pictorial device used to express a number of logical propositions. These pictures are commonly used as a teaching device to logically represent the inclusion or exclusion of sets with certain properties. However, used as such, the diagrams are usually expressed as two or three circles (which are called 2-set and 3-set Venn diagrams, respectively) which can represent at most eight logical areas.

The main thrust of the book is to show how it is possible to increase the number of propositions which can be expressed by such a diagram *ad infinitum*.

Before this aim is reached, a rich history of logical diagrams is given. There is a detailed historical and biographical account of John Venn's work on these diagrams including how the Venn diagrams superseded the Euler diagram and Venn's idea for expansion into 4-set and 5-set diagrams (which can represent 16 and 32 logical areas, respectively). Many other personalities are introduced, including Samuel Dodgson (a.k.a. Lewis Carrol), whom the author explained, "lost his courage" after creating only a 5-set diagram.

In order to expand to arbitrary n-set diagrams, one must look beyond the traditional circles. Venn himself constructed the 4-set diagram with ellipses, but even these cannot be further extended. Carrol tried to extend the number of areas using rectangles, but some of the areas which should be intersecting are in fact disconnected. Venn and others attempted diagrams with irregular shapes and managed a 5-set diagram, but this also could not easily be extended. The problem hinged on finding the right shape to represent the regions.

The author's solution projects the diagram onto a sphere. The ball can be split on its perpendicular axes to give a 3-set Venn diagram and then mutually intersecting wavy lines crossing the equator complete the diagram. The 4-set diagram of this type looks like a tennis ball. This diagram, known as the Edwards-Venn diagram, can be easily projected onto two dimensions. This two dimensional image can be seen as intersecting "cogwheels" with increasing numbers of "teeth", hence the title of the book.

The two dimensional Edwards-Venn diagrams are topologically equivalent to diagrams previously (and unknown to the author at the time) discovered by Branko Grünbaum. Grünbaum used polygons with increasing number of sides to represent the logical areas. The author found these diagrams to be unsatisfactory due to the speed at which the polygons converge on each other.

Along the same lines of thinking as the author's solution, Smith created an extendable set of Venn diagrams using a family of sine curves. An n-set diagram of this type is given by the formula $y = \sin(2^i x)/2^i$ where $x \in [0, 2\pi]$ and $0 \leq i \leq n-2$. Many curves in these diagrams will intersect in the same place unlike the Edwards-Venn diagrams, where only two curves can intersect at any one point.

Other algorithms for producing Edwards-Venn diagrams are also discussed, including those with rotational symmetry. One can enumerate all completely symmetrical n-set Venn diagrams using a computer program.

Applications of these new Venn diagrams to other areas of mathematics is the other major topic of the book. In connection with communications theory and combinatorics a Gray code interpretation of the Edwards-Venn diagrams is given. The Gray code is a sequence of binary numbers in which only one digit is changed in each successive pair. The areas of general Venn diagrams can be thought of as a binary numbers relating to the truth or falsity of the logical propositions. The Edwards-Venn diagram has the property that every boundary between logical areas also represents a change in only one digit in the binary representation.

In connection with graph theory, the two dimensional Edwards-Venn diagrams can be interpreted as planar representations of hypercubes. In particular, the dual graph of a Venn diagram is a maximal planar subgraph of a Boolean cube.

The book in general is geared towards non-mathematicians. The mathematics is very light and generally well-explained. There are many insightful and intriguing diagrams, including examples of Venn diagrams in national flags. In addition to the biographical history of the persons involved, the author adds his own story of his developing enthusiasm for the problem and how the solution was discovered. It can be interesting for scientists and non-scientists alike to witness how these discoveries are made.

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Entry for the Table of Contents: