Sandra.

HOD in ~~...~~ the models w/ Woodin cardinals.

thm. (Sargsyan, Woodin) for a cone of x:

$$\mathrm{HOD}^{M_n(x,g)} = M_n(\mathcal{M}_\infty, \Lambda),$$

where g is $Col(\omega, <\kappa)$-gen./ $M_n(x)$, κ the least ~~...~~ inaccessible of $M_n(x)$.

$\mathcal{M}_\infty$ = dir lim. model, Λ part. iteration strategy for $\mathcal{M}_\infty$.

$\mathcal{M}_\infty$ = dir lim of iterates of $(M_{n+1} | \delta_0 + \omega)^{M_{n+1}}$.
cf. later.

want to define an internal $\mathcal{M}_\infty$ in $M_n(x,g)$.
fix $x \geq_T M_{n+1} | (\delta_0 + \omega)^{M_{n+1}}$.

def. a ctbl premouse N is n-suitable iff

(1) $\mathcal{W} \models ZFC^-$, $\mathcal{W} \cap OR = \sup_{i<\omega} (\delta_i^{+i})^{\mathcal{W}}$

(2) $\mathcal{W} \models \delta$ is the only woodin

(3) for every cutpoint $\gamma < \delta$ of $\mathcal{W}$,

$S_{n+1}(\mathcal{W}|\gamma) \cancel{} = \mathcal{W}|\gamma^{+\mathcal{W}}$.

$\Uparrow$

$S_{n+1}(x) = M_{n+1}(x) \cap \mathbb{R}$ write $\delta^{\mathcal{W}}$ for the woodin.

(4) $\mathcal{W} = S_n(\mathcal{W}|\delta) | (\delta^{+\omega})^{S_n(\mathcal{W}|\delta)}$

example : $M_{n+1} | (\delta_0^{+\omega})^{M_{n+1}}$.

would like to define for every $s \in [OR]^{<\omega}$, $k<\omega$,

$$T_{s,k}^{\mathcal{W}} = \{ (t, \ulcorner\varphi\urcorner) \in [(\delta^{\mathcal{W}})^{+k\,\mathcal{W}}]^{<\omega} \times \omega :$$

$$\varphi \ \Sigma_1$$

$$M_n(\mathcal{W}|\delta^{\mathcal{W}}) \models \varphi(t,s) \}$$

$$\gamma_s^{\mathcal{W}} = \mathrm{Hull}_1^{\mathcal{W}}(\{T_{s,k}^{\mathcal{W}} : k<\omega\}) \cap \delta^{\mathcal{W}}$$

$$H_s^{\mathcal{W}} = \mathrm{Hull}_1^{\mathcal{W}}(\gamma_s^{\mathcal{W}} \cup \{T_{s,k}^{\mathcal{W}} : k<\omega\}).$$

<u>defn.</u> $\mathcal{F} = \{ H_s^{\mathcal{W}} : (\mathcal{W}, s) \in I \}$,

where $I = \{ (\mathcal{W}, s) :$ $\mathcal{W}$ is n-suitable,

W is strongly s-iterable }

solution (Sargsyan)

shrink the dir lim system and consider P-const.

$G_\kappa = \{ W \in M_n(x)|\kappa : W$ is a-suit.,

$M_n(x) \models$ " for some cutpoint η,

$\delta^W = \gamma^+$, $M_n(x)|\eta$ is

gen. / N for the δ^W-gen.

version of the extender algebra }

lem. let W be a app. it. n-suit.

premouse s.t. $W \in M_n(x)|\kappa$.

there is then an it. m of W s.t.

$m \in G_\kappa$, and

$$P^{M_n(x)}(m|\delta^m) = M_n(m|\delta^m).$$

in the def. of $T^{W}_{s,k}$ on p. 2,

repl. $M_n(-)$ by $\wp^{M_n(x)}(-)$.

$I_k = \{ (N, s) : N \in G_k ,\ s \in [OR]^{<\omega},$

$M_n(x) \models$ "N is strgly s-it. below k" $\}$

let $\mathcal{M}_\infty$ = dir lim of $\{ H^{N}_{s} :$

$(N, s) \in I_k \}$, with the ~~maps~~

can. maps between them.

let ~~π_i~~ $\pi^{*}_{(N,s),(M,t)} : H^{N}_{s} \longrightarrow H^{M}_{t}$

$\pi^{N}_{(N,s),\infty} : H^{N}_{s} \longrightarrow \mathcal{M}_\infty$.

$\hat{\mathcal{M}}_\infty = M_n(\mathcal{M}_\infty | \delta_\infty)$, $\hat{\pi}_{(N,s),(M,t)}$ ...

$\hat{\pi}_{(N,s),\infty}$ ...

let κ_∞ = the least inacc. at δ_∞ in $\hat{\mathcal{M}}_\infty$

let H be $\mathrm{Col}(\omega, <\kappa_\infty)$-gen. / $\hat{\mathcal{M}}_\infty$

<u>derived model resemblance</u> (Woodin)

let $\mathcal{W}$ be n-suitable, $s \in [OR]^{<\omega}$,

$(\mathcal{W}, s) \in \mathcal{I}_n$.

let $\bar{\xi} < \gamma_s^{\mathcal{W}}$, $\xi = \pi^*_{(\mathcal{W},s),\infty}(\bar{\xi})$,

$t \in [OR]^{<\omega}$.

let $\varphi(v_0, v_1, v_2)$ be a fmla in the lang of set thy.

TFAE.

(a) $\hat{\mathcal{M}}_\infty[H] \models \varphi(\mathcal{M}_\infty, \xi, t^*)$,

where $t^* = \{\alpha^* : \alpha \in t\}$,

$\alpha^* = \pi^*_{(\mathcal{M},s),\infty}(\alpha)$,

for $(\mathcal{M}, s) \in \mathcal{I}_n$,

$\alpha \in$

(b) $M_n(x, g) \models$ "there is a finite stack on $\mathcal{W}$ with last model $\mathcal{M}$ s.t. whenever $\mathcal{R} \in G_n$ is the last model of a finite stack on $\mathcal{M}$, then

$\varphi(\mathcal{R}, \pi^*_{(\mathcal{W},s),(\mathcal{R},s)}(\bar{\xi}), t)$."

<u>lem</u>. $V_{\delta_\infty}^{HOD^{M_n(x)[g]}} = V_{\delta_\infty}^{M_\infty}$.

<u>crucial lem</u>. $HOD^{M_n(x,g)} = M_n(A)$,

some $M_n(x,g)$ definable $A \subset \omega_2^{M_n(x,g)}$.

let $\mathbb{W}$ be the Vopěnka algebra in $M_n(x,g)$, G_x $\mathbb{W}$-gen. / $HOD^{M_n(x,g)}$, adding x. $\mathbb{P} = \mathbb{W} \times Col(\omega, <\kappa)$.

WTS: $HOD^{M_n(x,g)} = M_n(\mathbb{P})$.

let λ be the least inacc. of $M_n(x)$ above κ.

<u>claim 1</u>. $M_n(\mathbb{P}) | \lambda \subset HOD^{M_n(x,g)}$.

proof: let G be $Col(\omega, \omega_2^{M_n(x,g)})$ / gen. over $M_n(x,g)$. Compare $L[E](\mathbb{P})^{M_n(x,g)[G]}$ and $M_n(\mathbb{P})$.

$L[E](\mathbb{P})^{M_n(x,g)[G]} \cap HOD_{\mathbb{P}}^{M_n(x,g)[G]} \subset HOD_{\mathbb{P}}^{M_n(x,g)} = HOD^{M_n(x,g)}$

they coiterate to the same thing.

as G_x is $\mathbb{W}$-g. / $\mathrm{HOD}^{M_n(x,g)}$, $\overline{\overline{\mathbb{W}}} < \lambda$,

G_x is $\mathbb{W}$-g. over $M_n(\mathbb{P})|\lambda$.

<u>Claim 2.</u> $M_n(\mathbb{P})|\lambda\,[G_x] = M_n(x)|\lambda$.

"$\subset$": $G_x \in M_n(x)$.

"$\supset$": constr. $L[E](x)^{M_n(\mathbb{P})|\lambda}[G_x]$.

<u>Claim 3.</u>

$$M_n(\mathbb{P})[G_x] = M_n(x).$$

via P-constr.

<u>Claim 4.</u> $M_n(\mathbb{P}) \subset \mathrm{HOD}^{M_n(x,g)}$.

<u>Claim 5.</u> $M_n(\mathbb{P})[G_x] = \mathrm{HOD}^{M_n(x,g)}[G_x]$.