AXIOMATIC SET THEORY 1
SOSE 2019

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This is an introductory course to set theory. One of the aims of the course is to establish the independence of the Continuum Hypothesis from the usual axioms of set theory. Concepts introduced in the lectures will be further studied and examined during the proseminar, "Introductory Seminar in Set Theory" which is held Thursdays 9:45 - 11:45 at the KGRC.

The exam will be oral. Tentative dates to take the exam are 03.07.; 08.07.; 29.07. and 05.09. at 14:00. If you would like to take the exam in one of those dates, please send me an email at least 3 days in advance.

The classes are **Fridays, 11:30 - 14:30** in the KGRC seminar room.

Lecture 1, 01.03.: We discussed the axiomatic system ZFC, introduced the notion of an ordinal, established some basic properties and the Burali-Forti paradox.

Lecture 2, 07.03.: We showed that initial segments of the class of all ordinals are ordinals and introduced the notion of an order type; ordinal arithmetic and transfinite induction and recursion; cardinals; the theorem of Hartogs; the aleph function; We proved that AC is equivalent to the statement that for every two sets $x, y(x \prec y \vee y \prec x \vee x = y)$; CH and GCH. The matrial can be found in Section I.7. of [3] and sections I.11., I.12 of [4]

Lecture 3, 15.03.2019 cofinality; theorem of König; We stated the theorem of Easton; set-like and well-founded relations; transfinite induction and transfinite recursion of set-like and well-founded relations; $\text{rank}_{R,A}(x)$;

Lecture 4, 22.03.2019 sufficient condition for well-foundedness; rank of an ordinal; the class of well-founded sets; $R(\alpha)$; Mostowski collpase; uniqueness of transitive $\in$-models.

Lecture 5, 29.03.2019 Δ_0-formulas and their absolutenetss; relative interpretations; informal remarks on consistency proofs; consistency of foundation; examples of absolute notions; upwards and downwards absoluteness;

Lecture 6, 05.04. Reflection theorems; the Constructible sets; L-rank; ZF in L;

Lecture 7, 12.04. The axiom of constructibility; characterization of trnasitive set modelts of $ZF - P$ which satisfy constructibility;

lecture 8, 03.05. Infinitary combinatorics; Δ-system lemma; a.d. families of cardinality 2^κ; Martin's Axiom; Lemma of Solovay; MA implies that the continuum is regular;

Lecture 9, 10.05.2019 MA implies add($\mathcal{M}$) $> \kappa$; MA implies add($\mathcal{N}$) $> \kappa$; the method of forcing; $\mathbb{P}$-names; generic extensions; the forcing language; the Truth and Definability Lemmas; ZFC in $M[G]$.

Lecture 10, 31.05.2019 The forcing star relation; proofs of the Truth and Definability Lemmas; preservation of cardinals under ccc posets; approximation lemma;

Lecture 11, 07.06.2019 nice names; counting nice names; models of $\neg$CH; preservation of cardinals $\geq \theta$; Approximation lemma for θ-cc posets; θ-closure; models where GCH fails above $\aleph_0$;

Lecture 12, 14.06.2019 product forcing; proof of the theorem of Easton;

Lecture 13, 21.06.2019 top down approach; iterated forcing; consistency of MA and $\neg$CH;

References

[1] T. Jech *Set theory.* The third millennium edition, revised and expanded. Springer Monographs in Mathematics. Springer-Verlag, Berlin, 2003. xiv+769 pp

[2] L. Halbeisen *Combinatorial set theory. With a gentle introduction to forcing.* Springer Monographs in Mathematics. Springer, London, 2012. xvi+453 pp.

[3] K. Kunen *Set theory*, Studies in Logic (London), 34. College Publications, London, 2011, viii+401 pp.

[4] K. Kunen *The foundations of Mathematics* London : College Publ. ; 2012 ; Rev. ed..

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