PROSEMINAR AXIOMATIC SET THEORY I (S2019): 11.03.2018

Exercise 1: Let α, β, γ be ordinals. Show that:

(1) $(\alpha + \beta) + \gamma = \alpha + (\beta + \gamma)$
(2) $(\alpha \cdot \beta) \cdot \gamma = \alpha \cdot (\beta \cdot \gamma)$
(3) $\alpha \cdot (\beta + \gamma) = \alpha \cdot \beta + \alpha \cdot \gamma$.

Exercise 2: Let γ be a limit ordinal. Show that the following are equivalent:

(1) $\forall \alpha, \beta < \gamma (\alpha + \beta < \gamma)$
(2) $\forall \alpha < \gamma (\alpha + \gamma = \gamma)$
(3) $\forall X \subseteq \gamma (\mathrm{type}(X) = \gamma \vee \mathrm{type}(\gamma \setminus X) = \gamma)$

Exercise 3: Prove the *uniqueness* of the presentation in the Cantor Normal Form Theorem.

Exercise 4: For any set x:

(1) $x \subseteq \mathrm{trcl}(x)$
(2) $\mathrm{trcl}(x)$ is a transitive set
(3) If $x \subseteq t$ and t is transitive, then $\mathrm{trcl}(x) \subseteq t$.
(4) If $y \in x$ then $\mathrm{trcl}(y) \subseteq \mathrm{trcl}(x)$.