AXIOMATIC SET THEORY 1
SS 2016

VERA FISCHER

The course will cover Chapters II and IV of [3] and so can be viewed as an introduction to forcing. One of our goals is to establish the independence of the continuum hypothesis from the usual axioms of set theory. Other good sources are [2] and [1], however our main reference will be [3].

Some basic knowledge of cardinal and ordinal arithmetic (see Chapter I of [3]) will be assumed. Detailed information on the material covered during the semester will be regularly given here.

The exam will be oral.

The classes are on **Mondays, 8:30am-10:45am** in the KGRC seminar room.

Lecture 1, 07.03.: We made a review of the material covered in Chapter I of [3], up to Theorem I.9.11 (Transfinite Recursion on Well-founded Relations).

Lecture 2, 14.03.: We discussed the notion of a rank, as well as the Mostowski collapsing function - material corresponding to Section 9 of [3].

Lecture 3, 04.04.: We discussed hereditarily transitive sets, the Downward-Löwenheim-Skolem-Tarksi Theorem, the notions of definability and absoluteness (see sections I.15 and I.16 of [3]).

Lecture 4, 11.04.: We covered [3, Section II.2].

Lecture 5, 18.04.: We continued our discussion of absoluteness and covered section II.4 of [3].

Lecture 6, 25.04. We covered the reflection theorems from section II.4 of [3] and started our discussion of the Constructible Universe.

Lecture 7, 02.05. We covered section II.6 of [3] up to and including Lemma II.6.28.

Lecture 8, 09.05. We proved the Delta System Lemma, introduced Solovay's almost disjoint set partial order and proved that under MA 2^ω is regular.

Lecture 9, 23.05. We showed that MA(κ) implies each of the following: add($\mathcal{M}$) $> \kappa$ and add($\mathcal{N}$) $> \kappa$. In addition, we proved that MA(ω_1) implies that any product of ccc spaces is ccc and introduced the notion of a Suslin line.

Lecture 10, 30.05. MA(ω_1) impies SH; We started also our discussion of generic extensions and covered the material from Section IV.1. and Section IV.2. (up to Lemma IV.2.26.) of [3].

Lecture 11 and 12: We proved the Truth and Definability Lemmas, and covered section IV.3. up to and including Corollary IV.3.14

Lecture 13, 20.06. Complete and dense embeddings; maximal principle; θ-cc and λ-closure.

Lecture 14, 27.06. Failure of GCH above $\aleph_0$; generalized finite iterations and preservation of κ-cc under such iterations.

References

[1] T. Jech *Set theory.* The third millennium edition, revised and expanded. Springer Monographs in Mathematics. Springer-Verlag, Berlin, 2003. xiv+769 pp

[2] L. Halbeisen *Combinatorial set theory. With a gentle introduction to forcing.* Springer Monographs in Mathematics. Springer, London, 2012. xvi+453 pp.

[3] K. Kunen *Set theory*, Studies in Logic (London), 34. College Publications, London, 2011, viii+401 pp.

Kurt Gödel Research Center, University of Vienna, Währingerstrasse 25, 1090 Vienna, Austria

E-mail address: vera.fischer@univie.ac.at