AXIOMATIC SET THEORY 1
SS 2017

VERA FISCHER

This is an introductory course to set theory. Among the topics that it covers are Gödel's constructible universe, some infinitary combinatorics, Martin's axiom and its effects on the ideals of meager and measure zero sets. In addition, we will establish the independence of the Continuum Hypothesis from the usual axioms of set theory.

The exam will be oral.

The lectures will be taking place **Wednesdays from 11:30 to 13:50** in the KGRC lecture room (room 101).

Lecture 1, 08.03.: overview; the axioms of set theory; ordinals;
Lecture 2, 15.03.: well-founded and set-like relations; transfinite recursion; rank functions;
Lecture 3, 22.03.: the class of wellfounded sets; Mostowski collapse; König's Lemma;
Lecture 4, 29.03. cardinal exponentiation under GCH; hereditarily transitive sts; Δ_0-formulas;
Lecture 5, 05.04. consistency of foundation; absolutenss;
Lecture 6, 26.04. reflection theorems; the constructible universe;
Lecture 7, 03.05. L is a (class) model of $ZFC + GCH$;
Lecture 8, 10.05. We introduced the notion of κ-a.d. families and proved that there is always a κ-mad family (κ-maximal almost disjoint) of size 2^κ; We proved the Δ-System lemma; We defined Martin's Axiom (MA), as well as $MA(\kappa)$. We proved that $MA(\omega)$ is true and used the Cohen partial order to show that $MA(\mathfrak{c})$ is false. In addition, we defined the almost disjointness number $\mathfrak{a}$ and the almost disjointness partial order $\mathbb{P}_{\mathcal{A}}$. We defined the notion of being σ-centered and proved that σ-centered posets are ccc.
Lecture 9, 17.05. We proved: MA implies that $\text{cof}(2^\omega) = 2^\omega$; MA implies that $\text{add}(\mathcal{M}) = \mathfrak{c}$; MA implies that $\text{add}(\mathcal{N}) = \mathfrak{c}$.
Lecture 10, 24.05. We studied the effect of Martin's Axiom on the product of ccc topological spaces and showed that MA(ω_1) implies the Suslin Hypothesis. We also started our discussion of forcing and formulated the truth and definability Lemmas.
Lecture 11, 31.05. ZFC in generic extensions; introduction of the forcing star relation;

Lecture 12, 07.06. With the help of the focring star relation we proved the truth and defianbility Lemmas.

Lecture 13, 14.06. We discussed the problem of preservation of cardinals in generic extensions; models of $\neg$CH, as well as generic extensions in which GCH is violated above $\aleph_1$.

Lecture 14, 21.06. We made an overview of all the material covered throughout the semester.

On the last day of classes, 28.06., there will be exams. If you want to take the exam at a later date, please send me an E-mail.

References

[1] T. Jech *Set theory.* The third millennium edition, revised and expanded. Springer Monographs in Mathematics. Springer-Verlag, Berlin, 2003. xiv+769 pp

[2] L. Halbeisen *Combinatorial set theory. With a gentle introduction to forcing.* Springer Monographs in Mathematics. Springer, London, 2012. xvi+453 pp.

[3] K. Kunen *Set theory*, Studies in Logic (London), 34. College Publications, London, 2011, viii+401 pp.

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