MODEL THEORY S2018

VERA FISCHER

Content:

Two of the main goals of the course is to provide proofs of the Theorems of Baldwin-Lachlan and Morley. The course will be self-contained and will assume only knowledge in logic covered by all standard bachelor level courses.

Lecture 1 (09.03.2018): We made a detailed review of material usually covered in bachelor level classes in logic (chapters 1 and 2 of [4]).

Lecture 2 (16.03.2018): We proved the Separation theorem and gave characterization of theories axiomatizable with universal sentences (see Lemma 3.1.1, Corollary 3.1.5. of [4]).

Lecture 3 (23.03.2018): We made a review of standard material regarding complete theories with countable models. In particular we looked at ω-saturated models, atomic models, $\aleph_0$-categoricity. The material can be found in Section 3.3 of [2]. An alternative presentation can be found in [4, Sections 4.1,4.3, 4.5].

Lecture 4 (13.04.2018): We introduced the notion of κ-categoricty, indiscernibles, Ehrenfeucht-Mostowski type, proved the Sandard Lemma and discussed the number of types realized by structures generated by indiscernibles. We gave a characterization of theories with quantifier elimination. The material can be found in [4]: Theorem 3.2.5 and Section 5.1 up to and including Lemma 5.1.6.

Lecture 5 (20.04.2018): Skolem theories; ω-stable theories; The material corresponds to Corollary 5.1.9. and Section 5.2. of [4].

Lecture 6 (27.04.2018): Prime and Constructible Extensions; Theorem of Lachlan. The material corresponds to Section 5.3 up and including Corrollary 5.3.7. and Theorem 5.4.1. of [4].

Lecture 7 (04.05.2018): Morley Downwards; ω-homogenous structures; Vaughtian-Pairs and (κ, λ)-models. The covered material corresponds to Corollary 5.4.2, Lemma 5.5.3., Definition 5.5.1 and Corollary 5.5.4. of [4].

Lecture 8 (11.05.2018): We proved the theoram of Vaught (Theorem 5.5.2 of[4]), as well as Corollaries 5.5.4 and 5.5.5. of the same book. We looked at the algebraic closure operator and discussed some of its basic properties.

Lecture 9 (18.05.2018): We discussed strongly minimal and independent sets. The material can be found on Pages 79 and 81 of [4], as well as Lemma 6.1.4 of [1].

Lecture 10 (25.05.2018): We continued our discussion of strongly minimal formulas and their existence. More precisely, we looked at the following: dimension of a subset of a strongly minimal set; two models of a strongly minimal theory are isomorphic if and only if they are of the same dimension; strongly minimal theories are κ-categorical for each $\kappa \geq \aleph_1$; If $\mathfrak{M}$ is a model for a strongly minimal theory, then there is a minimal formula for $\mathfrak{M}$. The matrial corresponds to Lemma 6.1.9 - Lemma 6.1.13 of [1].

Final Grade:

The course grade will be based on an oral examination. For now, we have the following three possibilities to take the final exam: Monday, 25th of June at 2pm; Wednesday, 27th of June at 10am and Friday 29th of June at 3pm. If you plan to take the exam at any of these three dates, please send me an Email.

When and where: The lectures are taking place Fridays from 15:00 to 16:30 in the lecture room of the Kurt Gödel Research Center (Währinger Straße 25, 2nd floor, 02.101).

References

[1] Marker, David *Model theory. An introduction.* Graduate Texts in Mathematics, 217. Springer-Verlag, New York, 2002. viii+342 pp. ISBN: 0-387-98760-6

[2] Goldstern Martin; Judah, Haim *The incompleteness phenomenon.* A. K. Peters, Natick, Massachusetts, ISBN-10: 1568810938

[3] Hodges, Wilfrid *A shorter model theory.* Cambridge University Press, Cambridge, 1997. x+310 pp. ISBN: 0-521-58713-1

[4] Tent, Katrin; Ziegler, Martin *A course in model theory.* Lecture Notes in Logic, 40. Association for Symbolic Logic, La Jolla, CA; Cambridge University Press, Cambridge, 2012. x+248 pp. ISBN: 978-0-521-76324-0

Kurt Gödel Research Center, University of Vienna, Währingerstrasse 25, 1090 Vienna, Austria

E-mail address: vera.fischer@univie.ac.at