

## AXIOMATIC SET THEORY I: 09.03.2018

**Exercise 1.** Justify the existence of each of the following:

- (1) Let  $A, B$  be sets. Then  $A \times B = \{(x, y) : x \in A, y \in B\}$  is a set.
- (2) Let  $R$  be a set. Then  $\text{dom}(R) = \{x : \exists y((x, y) \in R)\}$  and  $\text{ran}(R) = \{y : \exists x((x, y) \in R)\}$  are sets.
- (3) Let  $A, B$  be sets. Then the collection of all functions with domain  $A$  and range contained in  $B$  is a set.

**Exercise 2.** Prove that for all ordinals  $\alpha, \beta, \gamma$ :

- (1) if  $\alpha \leq \beta$ , then  $\alpha + \gamma \leq \beta + \gamma$ ,
- (2)  $(\alpha + \beta) + \gamma = \alpha + (\beta + \gamma)$

**Exercise 3.** Show that ordinal multiplication is not commutative.

**Exercise 4.** Let  $\alpha, \beta, \gamma$  be ordinals. Show that

- (1)  $(\alpha \cdot \beta) \cdot \gamma = \alpha \cdot (\beta \cdot \gamma)$
- (2)  $\alpha \cdot (\beta + \gamma) = \alpha \cdot \beta + \alpha \cdot \gamma$

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