Homework 3, due: Oct 13, 11:30 am

(1) Let $A \subset \mathbb{R}$ be a λ-measurable set with $\lambda(A) < \infty$ and $\varepsilon > 0$. Prove that there exists a finite collection of disjoint open intervals $I_1, \ldots, I_n$ such that if $U = \bigcup_{i=1}^n I_i$ then $\lambda((A \setminus U) \cup (U \setminus A)) < \varepsilon$.

(2) Prove the statement from the lecture: if X is a metric space and μ is a finite measure on $\mathcal{B}_X$ then the collection of μ measurable sets E for which $\mu(E) = \inf\{\mu(U) : U \supset E, U \text{ is open}\} = \sup\{\mu(F) : F \subset E, F \text{ is closed}\}$ holds form a σ-algebra.

(3) Construct a subset C of the reals which contains no intervals but $\lambda(C) > 0$.