Homework 4, due: Oct 20, 11:30 am

(1) Show that every monotone function $f : \mathbb{R} \to \mathbb{R}$ is Borel measurable.
(2) Prove the following statement from the lecture: if μ is a complete measure on a space $(X, \mathcal{M})$, the functions $f_n : X \to \mathbb{R}$ are μ-measurable and for some $f : X \to \mathbb{R}$ we have that $f_n(x) \to f(x)$ holds μ-almost everywhere then f is μ-measurable.
(3) Let $A \subset \mathbb{R}$ be an arbitrary set. Show that if $\lambda^*(A) > 0$ then there exists a nonempty open interval I such that

$$\lambda^*(I \cap A) > 0.99 \cdot \lambda(I).$$