Homework 5, due: Nov 1, 11:30 am

(1) (a) Construct an example of $f_n : \mathbb{R} \to \mathbb{R}$ functions, with $f_n \to f$ uniformly, but $\int f_n d\lambda \not\to \int f d\lambda$.
    (b) Let $(X, \mathcal{M}, \mu)$ be a *finite* measure space and $f_n : X \to \mathbb{R}$ be measurable functions, and suppose that $f_n \to f$ uniformly. Prove that $\int f_n d\lambda \to \int f d\lambda$.

(2) Let $H \subset \mathbb{R}$ be measure zero. Prove that H can be translated into the irrationals, i. e., there exists an $x \in \mathbb{R}$ such that for every $h \in H$ the number $x + h$ is irrational!

(3) Let $f_n : [0, 1] \to \mathbb{R}$ be a sequence of continuous functions. Show that the set $\{x : \text{the sequence } (f_n(x))_{n=1}^{\infty} \text{ converges}\}$ is $F_{\sigma\delta}$, that is, the countable intersection of F_σ sets.