MATH6280 A - Measure Theory
Midterm Examination, Sample

(1) (a) Define the notion of a monotone class.
    (b) Decide whether the following families form a monotone class: $\mathcal{A} = \{I \subset \mathbb{R} : I \text{ is a closed interval}\}$ and $\mathcal{B} = \{I \subset \mathbb{R} : I \text{ is an interval}\}$.

(2) Let $(X, \mathcal{M}, \mu)$ be a measure space and $f : X \to \mathbb{R}$ be measurable. Prove that if for every $x \in X$ we have $f(x) > 0$ then $\int f d\mu > 0$.

(3) Let $H \subset [0, 1]$ be an arbitrary set. Prove that there exists a real $c \in \mathbb{R}$ such that $\lambda^*([0, c] \cap H) = \lambda^*(H)/2$.

(4) Let $\varepsilon > 0$ and suppose that for every n we have a λ measurable set $E_n \subset [0, 1]$ such that $\lambda(E_n) > \varepsilon$. Prove that
    (a) for every k there exists an $x \in [0, 1]$ such that $x \in E_n$ for at least $k \in \mathbb{N}$ many n's,
    (b) there exists an $x \in [0, 1]$ such that $x \in E_n$ for infinitely many n's.