Simpliciality of vector-valued function spaces

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Based on the paper:

[KS] O.F.K. Kalenda and J. Spurný: On simpliciality of vector-valued function spaces, arXiv:2501.12876

The scalar case

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- H contains constants $\Rightarrow M_t(H) \subset M_1(K)$

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Remark
K metrizable $\Rightarrow$ (μ H-boundary $\Leftrightarrow$ μ carried by $\operatorname{Ch}_H K$).

Theorem

Let $H \subset C(K, \mathbb{F})$ be a function space and let $\psi \in H^*$. Then $M_\psi(H)$ contains an *H*-boundary measure.

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History of proofs

- H separates points and contains constants, $\mathbb{F} = \mathbb{R}$
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- Represent $\psi \in H^*$ via integration with respect to E^*-valued measures on K.

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Remark

In the scalar case ($E = \mathbb{F}$) both ways coincide.

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Singer's Theorem

$C(K, E)^* = M(K, E^*)$, the space of regular E^*-valued Borel measures on K of bounded variation
($\int \chi_A \cdot x \,\mathrm{d}\mu = \mu(A)(x)$ for $A \subset K$ Borel, $x \in E$, $\mu \in M(K, E^*)$)

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Representation theorem [Saab & Talagrand, 1985]

$\forall \psi \in H^* \exists \mu \in M_\psi(H)$ s.t. $|\mu|$ is H_w-boundary

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$$L_{rep}(H, E) = \{U \in L(H, E) \; ; \; \exists \mu \in M(K) \, \forall \boldsymbol{f} \in H \colon \; U(\boldsymbol{f}) = (B) \int \boldsymbol{f} \, \mathrm{d}\mu\}$$

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Theorem

$\exists \Upsilon : H_w^* \to L_{rep}(H,E)$, a bijective linear isometry such that

- Υ is w*-to-WOT continuous;
- $\Upsilon(\phi_{H_w}(t)) = \phi_H(t)$;
- $M_{\Upsilon(\psi)}(H) = M_\psi(H_w)$.

Case 2 - representing measures of operators II

Corollary

- Representing operators $H \to E$ is the same as representing functionals on H_w.
- $M_U(H) \neq \emptyset$ for each $U \in L_{rep}$. Hence, infimum in the definition of $\|\cdot\|_{rep}$ is attained.
- $M_U(H)$ contains a H_w-boundary measure for each $U \in L_{rep}$.

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H . . . a linear subspace of $C(K, E)$ separating points of K

Definition

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- $E = \mathbb{F} \Rightarrow$ weakly = vector

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Remark

$E = \mathbb{F} \Rightarrow A_c^w(H) = A_c^v(H) = A_c(H)$

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Theorem

TFAE

- H is vector simplicial;
- $A_c^v(H)$ is vector simplicial;
- $A_c^v(H)$ is functionally vector simplicial;
- H is weakly simplicial and $A_c^v(H) = A_c^w(H)$;
- the only H-boundary measure in $A_c^v(H)^\perp$ is 0.