EINFÜHRUNG IN DIE MATHEMATISCHE LOGIK WS 2016

VERA FISCHER

The course is an introduction to mathematical logic. We will cover the theorems of Gödel for Incompleteness, as well as some introductory model and set theory. Some knowledge from the course of Dr. Moritz Müller, "Grundbegriffe der mathematischen Logik ", see

http://www.logic.univie.ac.at/∼muellem3/teaching.html.

will be assumed. However, the course is **self-contained**. Our main references are listed below. Detailed information on the material covered during the semester will be regularly given here.

The exam will be oral.

The lectures will be taking place **Tuesdays and Fridays at 9:00am** in the KGRC, lecture room 101.

Lecture 1, 04.10.: inductive structures, sentential language, unique readability;
Lecture 2, 05.10.: truth assignments, conjunctive normal form, complete languages;
Lecture 3, 11.10.: first order languages, terms, formulas, substitution, models;
Lecture 4. 14.10: tautologies, the satisfaction relation, validity; we provided proofs (for most of) the axioms groups introduced in Lecture 5, that they represent valid formulas.
Lecture 5, 18.10: proof systems, derivations, the theorems of soundness and deduction; the notion of concsistency;
Lecture 6, 21.10: the theorems of generalization, introduction of ∀, resp. ∃ quantifiers; enumerability; compactness theorem for first order languages; complete simple extensions;
Lecture 7, 25.10: Henkinization of a theory, the theorem of completeness;
Lecture 8, 28.10: restriction (expansion) of a model; elementarily equivalent models; chain of models;
Lecture 9, 04.11. elementary submodels; Tarski-Vaught Criterion; elementary chains; elementary embeddings and characterizations;
Lecture 10, 08.11. joint consistency theorem; ultrapowers; theorem of Loś;

Lecture 11, 11.11. semantic compactness; non-standard models of arithmetic; n-types;
Lecture 12, 15.11. alternative proof of the fact that the theory of arithmetic has uncountably many non-isomorphic countable models; a complete theory with a unique up to isomorphism countable model (DLO); a complete theory with exactly three up to isomorphism countable models (DLO^+).
Lecture 13, 18.11. saturated models; saturated models are universal; characterization of theories with saturated models;
Lecture 14, 22.11. atomic models, characterization of atomic models;
Lecture 15, 25.11. ω-categorical theories; the never two theorem;
Lecture 16, 29.11. Axioms of Peano Arithmetic, basic theorems of PA; Gödel codes;
Lecture 17, 30.11. encoding of terms, formulas, derivations and theorems; term substitution in PA;
Lecture 18, 2.12. formula substitution in PA; Gödel Incompleteness Theorem; Gödel's Self-Referential Lemma; Undefinability of Truth;
Lecture 19, 6.12. The Axioms of Set Theory; justification of various definitions; well-founded relations;
Lecture 20, 13.12. ordinals; Burali-Forti's paradox; order type; ordinal addition and multiplication;
Lecture 21, 16.12. transfinite induction and recursion; theorem of Schröder-Bernstein; Cantor's diagonal element Lemma;
Lecture 22, 10.01. cardinals; axiom of choice; equivalent formulations
Lecture 23, 17.01. maximal principles;
Lecture 24, 20.01. cardinal arithmetic; cofinalities;

References

[1] H.-D. Ebbinghaus, J. Flum, W. Thomas *Einführung in die mathematische Logik* Springer, 2007.
[2] M. Goldstern, H. Judah *The incompleteness phenomenon*, A. K. Peters, 1998.
[3] K. Kunen *The Foundations of Mathematics*, Studies in Logic (London), 19. Mathematical Logic and Foundations. College Publications, London, 2009. viii+251 pp.
[4] K. Kunen *Set theory*, Studies in Logic (London), 34. College Publications, London, 2011, viii+401 pp.

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