THEORETICAL COMPUTER SCIENCE
WS 2017

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This is a master level course in theoretical computer science. We will cover central topics in recursion theory and computational complexity. **The lectures are taking place Tuesdays from 15:15 to 17:30 at the Seminar room of the KGRC (room 101).** The final exam will be oral.

Detailed information on the material covered during the semester, including relevant references to the literature, will be regularly given here. Our two main textbooks are listed below. Both of those textbooks are available for **free download** via the **University of Vienna Library**. The book of Herbert Enderton, *Computability theory. An introduction to recursion theory*, gives a detailed introduction to the subject of recursion theory and will be used in the beginning of the course. The book of Sanjeev Arora and Boaz Barak, *Computational complexity. A Modern Approach*, gives a comprehensive account of many interesting topics in computational complexity. Another excellent source is [3].

Lecture 1, 03.10.: We considered the non-mathematical notion of effective calculability, considered the undecidability of the Halting problem, the theorem of Kleene and the thesis of Church. Moreover we defined Turing machines, defined the class of Turing computable functions and formulated the Church-Turing thesis.

Lecture 2, 17.10.: We defined the classes of primitive recursive and general recursive functions, and studied many examples of such functions. The notions of register machine, as well as function computable by a register machine were defined. In addition, we gave a definition of computable functions. Most of the material can be found in Sections 1.2 and 2.1 of [1].

Lecture 3, 24.10.: We proved that every general recusrive function is register machine computable and considered a universal register machin which mimics every register machine. The material can be found in Sections 3.1 and 3.2 of [1].

Lecture 4, 31.10.: We considered the universal function, the diagonal function, the halting relation, the running time function and proved the normal form theorem. As a conlcusion we obtained that every register machine computable partial function is general recursive. Furthermore, we defined the notion of recursive enumerability, showed that the halting relation is not computable, but r.e. Finally, we proved Kleene's theorem and as an immediate corollary obtained that the complement of the halting relation is not r.e. The material can be found in sections 3.2 and 4.1 of [1].

Lecture 5, 07.11.: We established various properties of the recursively enumerable sets. In particular, we proved the following facts:

- the graph of a partial function is r.e. iff the function is computable;
- the range of a computable partial function is a r.e. set;
- a subset of $\mathbb{N}$ is r.e. iff it is the range of a total, computable function, or is empty;
- the set of indices of all total, unary, computable functions is not r.e.;
- the set of indices of all total, unary, computable functions is not co-r.e.;
- K is a complete r.e. set;

as well as the Parameter Theorem, and the Theorem of Rice. The material can be found in Sections 4.1 and 4.2 of [1].

Lecture 6, 14.11.: We studied the notion of Turing reducibility, leading to the quotient space of Turing degrees and the jump operator. The Lecture covered most of the material from Chapter 6 of [1].

Lecture 7, 21.11.: We discussed the arithmetic hierarchy and proved Gödel's theorem of incompleteness. The relevant material can be found in Chapter 5 of [1]. Also, we started our discussion of Gödel's second theorem of incompleteness, an outline of the proof of which will be given next time (for an independent presentation see [4]). The proof of Gödel's second incompleteness theorem, will **not** be required at the final exam.

Lecture 8, 05.12.: We proved Gödel's second theorem of incompleteness. We formalized the notion of a decision problem and introduced multitape Turing machines.

Lecture 9, 06.12.: We gave a detailed analysis on the work of a multitape Turing machine by defining the configuration graph of a TM and introduced computation tables for single tape Turing machines. In addition, we discussed the robustness of the computation model of TM. Finally, we intorduced the big-O and small-o notations, and defined the classes of polynomial time, simply exponential time and exponential time.

Lecture 10, 12.12.: We showed that PATH $\in \mathbb{P}$, constructed an UTM, proved the time hierarchy theorem, defined nondeterministic TM, abbr. NTM, and non-det. time classes. We showed that every NTM running in time $O(t(n))$ can be simulated by a det. TM running in time $2^{\mathrm{O}(t(n))}$, and discussed $\mathbb{P} \subseteq \mathbb{NP} \subseteq \mathbb{EXP}$.

Lecture 11, 09.01.: We defined deterministic and nondeterministic space complexity, proved the space hierarchy theorem, discussed relations between space and time complexity classes, proved the theorem of Savitch, and that every problem that can be decided in nondeterministic space $f(n)$, can be decided in determininstic space $f(n)$.

Lecture 12, 16.01.: We defined the problems SAT, CIRCUIT SAT and HAMILTON PATH, discussed the notions of reduction and completeness, proved that HAMILTON PATH is reducible to SAT, as well as the fact that CIRCUIT SAT is reducible to SAT.

Lecture 13, 23.01.: We proved that CIRCUIT VALUE is $\mathbb{P}$-complete, as well as that SAT is $\mathbb{NP}$ complete. We proved the theorem of Immerman-Szelepscényi, and concluded that nondeterministic space complexity classes are closed with respect to complements.

The final exam will be on the **30.01. at 15:15**. A second date for taking the exam is **15.02. at 9:00am.**

References

[1] H. Enderton *Computability theory. An introduction to recursion theory.* Elsevier/Academic Press, Amsterdam, 2011. xii+174 pp. ISBN: 978-0-12-384958-8.

[2] S. Arora, B. Barak *Computational complexity. A Modern Approach.* Cambridge University Press, Cambridge, 2009. xxiv+579 pp. ISBN: 978-0-521-42426-4.

[3] C. H. Papadimitriou *Computational complexity.* Addison-Wesley Publishing Company, Reading, MA, 1994. xvi+523 pp. ISBN: 0-201-53082-1

[4] M. Ziegler *Mathematische Logik.* Mathematik Kompakt. Birkhuser Verlag, Basel, 2010. viii+116 pp. ISBN: 978-3-7643-9973-3

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